

CHAPTER 6

Additional Topics in Trigonometry

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CHAPTER 6

Additional Topics in Trigonometry

Section 6.1 Law of Sines

- If ABC is any oblique triangle with sides a , b , and c , then the Law of Sines says

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

- You should be able to use the Law of Sines to solve an oblique triangle for the remaining three parts, given:

- (a) Two angles and any side (AAS or ASA)
(b) Two sides and an angle opposite one of them (SSA)

1. If A is acute and $h = b \sin A$:
(a) $a < h$, no triangle is possible.
(b) $a = h$ or $a \geq b$, one triangle is possible.
(c) $h < a < b$, two triangles are possible.
2. If A is obtuse and $h = b \sin A$:
(a) $a \leq b$, no triangle is possible.
(b) $a > b$, one triangle is possible.

- The area of any triangle equals one-half the product of the lengths of two sides times the sine of their included angle.

$$A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B = \frac{1}{2}bc \sin A$$

Vocabulary Check

1. oblique

2. $\frac{b}{\sin B}$

3. (a) Two; any; AAS; ASA

4. $\frac{1}{2}bc \sin A$; $\frac{1}{2}ab \sin C$; $\frac{1}{2}ac \sin B$

(b) Two; an opposite; SSA

1. Given: $A = 25^\circ$, $B = 60^\circ$, $a = 12$

$$C = 180^\circ - 25^\circ - 60^\circ = 95^\circ$$

$$b = \frac{a}{\sin A}(\sin B) = \frac{12}{\sin 25^\circ}(\sin 60^\circ) \approx 24.59 \text{ in.}$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{12}{\sin 25^\circ}(\sin 95^\circ) \approx 28.29 \text{ in.}$$

2. Given: $A = 35^\circ$, $B = 55^\circ$, $a = 18$

$$C = 180^\circ - 35^\circ - 55^\circ = 90^\circ$$

$$b = \frac{a}{\sin A}(\sin B) = \frac{18}{\sin 35^\circ}(\sin 55^\circ) \approx 25.71 \text{ mm}$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{18}{\sin 35^\circ}(\sin 90^\circ) \approx 31.38 \text{ mm}$$

3. Given: $B = 15^\circ$, $C = 125^\circ$, $c = 20$

$$A = 180^\circ - 15^\circ - 125^\circ = 40^\circ$$

$$a = \frac{c}{\sin C}(\sin A) = \frac{20}{\sin 125^\circ}(\sin 40^\circ) \approx 15.69 \text{ cm}$$

$$b = \frac{c}{\sin C}(\sin B) = \frac{20}{\sin 125^\circ}(\sin 15^\circ) \approx 6.32 \text{ cm}$$

4. Given: $B = 40^\circ$, $C = 110^\circ$, $c = 30$

$$A = 180^\circ - 40^\circ - 110^\circ = 30^\circ$$

$$a = \frac{c}{\sin C}(\sin A) = \frac{30}{\sin 110^\circ}(\sin 30^\circ) \approx 15.96 \text{ ft}$$

$$b = \frac{c}{\sin C}(\sin B) = \frac{30}{\sin 110^\circ}(\sin 40^\circ) \approx 20.52 \text{ ft}$$

5. Given: $A = 80^\circ 15'$, $B = 25^\circ 30'$, $b = 2.8$

$$C = 180^\circ - 80^\circ 15' - 25^\circ 30' = 74^\circ 15'$$

$$a = \frac{b}{\sin B}(\sin A) = \frac{2.8}{\sin 25^\circ 30'}(\sin 80^\circ 15') \approx 6.41 \text{ km}$$

$$c = \frac{b}{\sin B}(\sin C) = \frac{2.8}{\sin 25^\circ 30'}(\sin 74^\circ 15') \approx 6.26 \text{ km}$$

6. Given: $A = 88^\circ 35'$, $B = 22^\circ 45'$, $b = 50.2$

$$C = 180^\circ - 88^\circ 35' - 22^\circ 45' = 68^\circ 40'$$

$$a = \frac{b}{\sin B}(\sin A) = \frac{50.2}{\sin 22^\circ 45'}(\sin 88^\circ 35') \approx 129.77 \text{ yd}$$

$$c = \frac{b}{\sin B}(\sin C) = \frac{50.2}{\sin 22^\circ 45'}(\sin 68^\circ 40') \approx 120.92 \text{ yd}$$

7. Given: $A = 36^\circ$, $a = 8$, $b = 5$

$$\sin B = \frac{b \sin A}{a} = \frac{5 \sin(36^\circ)}{8} \approx 0.3674 \Rightarrow B \approx 21.6^\circ$$

$$C = 180^\circ - A - B \approx 180^\circ - 36^\circ - 21.6^\circ = 122.4^\circ$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{8}{\sin(36^\circ)} \sin(122.4^\circ) \approx 11.49$$

8. Given: $A = 60^\circ$, $a = 9$, $c = 10$

$$\sin C = \frac{c \sin A}{a} = \frac{10 \sin 60^\circ}{9} \approx 0.9623 \Rightarrow C \approx 74.2^\circ \text{ or } C \approx 105.8^\circ$$

Case 1

$$C \approx 74.2^\circ$$

$$B = 180^\circ - A - C \approx 45.8^\circ$$

$$b = \frac{a}{\sin A}(\sin B) \approx \frac{9 \sin 45.8^\circ}{\sin 60^\circ} \approx 7.45$$

Case 2

$$C \approx 105.8^\circ$$

$$B = 180^\circ - A - C \approx 14.2^\circ$$

$$b = \frac{a}{\sin A}(\sin B) \approx \frac{9 \sin 14.2^\circ}{\sin 60^\circ} \approx 2.55$$

9. Given: $A = 102.4^\circ$, $C = 16.7^\circ$, $a = 21.6$

$$B = 180^\circ - A - C = 180^\circ - 102.4^\circ - 16.7^\circ = 60.9^\circ$$

$$b = \frac{a}{\sin A}(\sin B) = \frac{21.6}{\sin 102.4^\circ}(\sin 60.9^\circ) \approx 19.32$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{21.6}{\sin 102.4^\circ}(\sin 16.7^\circ) \approx 6.36$$

10. Given:
- $A = 24.3^\circ$
- ,
- $C = 54.6^\circ$
- ,
- $c = 2.68$

$$B = 180^\circ - A - C = 101.1^\circ$$

$$a = \frac{c}{\sin C}(\sin A) = \frac{2.68 \sin 24.3^\circ}{\sin 54.6^\circ} \approx 1.35$$

$$b = \frac{c}{\sin C}(\sin B) = \frac{2.68 \sin 101.1^\circ}{\sin 54.6^\circ} \approx 3.23$$

11. Given:
- $A = 110^\circ 15'$
- ,
- $a = 48$
- ,
- $b = 16$

$$\sin B = \frac{b \sin A}{a} = \frac{16 \sin 110^\circ 15'}{48} \approx 0.31273 \Rightarrow B \approx 18^\circ 13'$$

$$C = 180^\circ - A - B \approx 180^\circ - 110^\circ 15' - 18^\circ 13' = 51^\circ 32'$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{48}{\sin 110^\circ 15'}(\sin 51^\circ 32') \approx 40.06$$

12. Given:
- $B = 2^\circ 45'$
- ,
- $b = 6.2$
- ,
- $c = 5.8$

$$\sin C = \frac{c \sin B}{b} = \frac{5.8 \sin 2^\circ 45'}{6.2} \approx 0.04488 \Rightarrow C \approx 2.57^\circ (2^\circ 34')$$

$$A = 180^\circ - B - C \approx 174.68^\circ (174^\circ 41')$$

$$a = \frac{b}{\sin B}(\sin A) \approx \frac{6.2 \sin 174^\circ 41'}{\sin 2^\circ 45'} \approx 11.97$$

13. Given:
- $A = 110^\circ$
- ,
- $a = 125$
- ,
- $b = 100$

$$\sin B = \frac{b \sin A}{a} = \frac{100 \sin 110^\circ}{125} \approx 0.75175 \Rightarrow B \approx 48.74^\circ$$

$$C = 180^\circ - A - B \approx 21.26^\circ$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{125 \sin 21.26^\circ}{\sin 110^\circ} \approx 48.23$$

14. Given:
- $A = 110^\circ$
- ,
- $a = 125$
- ,
- $b = 200$

$$A \text{ obtuse and } a < b \Rightarrow \text{No solution}$$

15. Given:
- $A = 76^\circ$
- ,
- $a = 18$
- ,
- $b = 20$

$$\sin B = \frac{b \sin A}{a} = \frac{20 \sin 76^\circ}{18} \approx 1.078$$

$$\text{No solution}$$

16. Given:
- $A = 76^\circ$
- ,
- $a = 34$
- ,
- $b = 21$

$$\sin B = \frac{b \sin A}{a} = \frac{21 \sin 76^\circ}{34} \approx 0.5993 \Rightarrow B \approx 36.8^\circ$$

$$C \approx 180^\circ - 76^\circ - 36.8^\circ \approx 67.2^\circ$$

$$c = \frac{a}{\sin A} \sin C = \frac{34}{\sin 76^\circ} \sin 67.2^\circ \approx 32.30$$

17. Given: $A = 58^\circ$, $a = 11.4$, $b = 12.8$

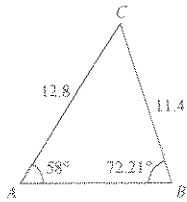
$$\sin B = \frac{b \sin A}{a} = \frac{12.8 \sin 58^\circ}{11.4} \approx 0.9522 \Rightarrow B \approx 72.21^\circ \text{ or } 107.79^\circ$$

Case 1

$$B \approx 72.21^\circ$$

$$C = 180^\circ - 58^\circ - 72.21^\circ = 49.79^\circ$$

$$c = \frac{a}{\sin A} (\sin C) = \frac{11.4}{\sin 58^\circ} (\sin 49.79^\circ) \approx 10.27$$

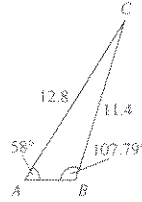


Case 2

$$B \approx 107.79^\circ$$

$$C \approx 180^\circ - 58^\circ - 107.79^\circ = 14.21^\circ$$

$$c = \frac{a}{\sin A} (\sin C) = \frac{11.4}{\sin 58^\circ} (\sin 14.21^\circ) \approx 3.30$$



18. Given: $A = 58^\circ$, $a = 4.5$, $b = 12.8$

$$a < h = b \sin 58^\circ$$

$$4.5 < 10.86$$

No solution

19. Area $= \frac{1}{2} ab \sin C$

$$= \frac{1}{2} (6)(10) \sin(110^\circ)$$

$$\approx 28.2 \text{ square units}$$

20. Area $= \frac{1}{2} ac \sin B$

$$= \frac{1}{2} (92)(30) \sin(130^\circ)$$

$$\approx 1057.1 \text{ square units}$$

21. Area $= \frac{1}{2} bc \sin A$

$$= \frac{1}{2} (67)(85) \sin(38^\circ 45')$$

$$\approx 1782.3 \text{ square units}$$

22. $A = 5^\circ 15'$, $b = 4.5$, $c = 22$

$$\text{Area} = \frac{1}{2} bc \sin A$$

$$= \left(\frac{1}{2}\right)(4.5)(22) \sin 5.25^\circ$$

$$\approx 4.529 \text{ square units}$$

23. Area $= \frac{1}{2} ac \sin B$

$$= \frac{1}{2} (103)(58) \sin 75^\circ 15'$$

$$\approx 2888.6 \text{ square units}$$

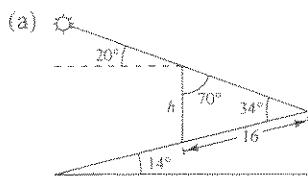
24. Area $= \frac{1}{2} ab \sin C$

$$= \frac{1}{2} (16)(20) \sin 85^\circ 45'$$

$$\approx 159.6 \text{ square units}$$

25. Angle $CAB = 70^\circ$

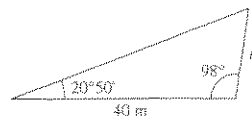
$$\text{Angle } B = 20^\circ + 14^\circ = 34^\circ$$



(b) $\frac{16}{\sin 70^\circ} = \frac{h}{\sin 34^\circ}$

(c) $h = \frac{16 \sin 34^\circ}{\sin 70^\circ} \approx 9.52 \text{ meters}$

26. (a)



$$20^\circ 50' \approx 20.83^\circ$$

(b) $A = 180^\circ - 98^\circ - 20.83^\circ = 61.17^\circ$ (or $61^\circ 10'$)

$$\frac{40}{\sin A} = \frac{h}{\sin (20^\circ 50')} \Rightarrow h = \frac{40 \sin (20^\circ 50')}{\sin 61.17^\circ}$$

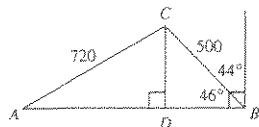
(c) $h \approx 16.2 \text{ m}$

$$27. \sin A = \frac{a \sin B}{b} = \frac{500 \sin(46^\circ)}{720} \approx 0.4995$$

$$A \approx 29.97^\circ$$

$$\angle ACD = 90^\circ - 29.97^\circ \approx 60^\circ$$

Bearing: S 60° W or (240° in plane navigation)

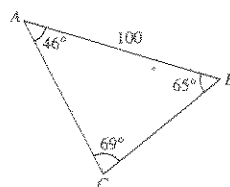


$$28. \text{ Given: } A = 74^\circ - 28^\circ = 46^\circ,$$

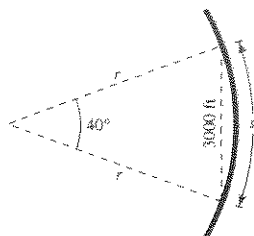
$$B = 180^\circ - 41^\circ - 74^\circ = 65^\circ, c = 100$$

$$C = 180^\circ - 46^\circ - 65^\circ = 69^\circ$$

$$a = \frac{c}{\sin C} (\sin A) = \frac{100}{\sin 69^\circ} (\sin 46^\circ) \approx 77 \text{ meters}$$



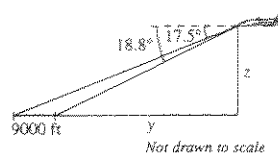
29. (a)



$$(b) r = \frac{3000 \sin[1/2(180^\circ - 40^\circ)]}{\sin 40^\circ} \approx 4385.71 \text{ feet}$$

$$(c) s \approx 40^\circ \left(\frac{\pi}{180^\circ} \right) 4385.71 \approx 3061.80 \text{ feet}$$

30. (a)



$$(b) \frac{x}{\sin 17.5^\circ} = \frac{9000}{\sin 1.3^\circ}$$

$$x \approx 119,289.1261 \text{ feet} \approx 22.6 \text{ miles}$$

$$(c) \frac{y}{\sin 71.2^\circ} = \frac{x}{\sin 90^\circ}$$

$$y = x \sin 71.2^\circ \approx 119,289.1261 \sin 71.2^\circ$$

$$\approx 112,924.963 \text{ feet} \approx 21.4 \text{ miles}$$

$$(d) z = 119,289.1261 \sin 18.8^\circ \approx 38,442.8 \text{ feet}$$

$$31. \angle ACD = 65^\circ$$

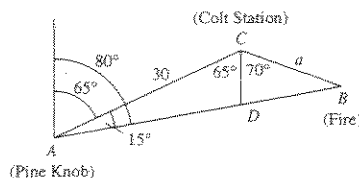
$$\angle ADC = 180^\circ - 65^\circ - 15^\circ = 100^\circ$$

$$\angle CDB = 180^\circ - 100^\circ = 80^\circ$$

$$\angle B = 180^\circ - 80^\circ - 70^\circ = 30^\circ$$

$$a = \frac{b}{\sin B} (\sin A) = \frac{30}{\sin 30^\circ} (\sin 15^\circ) \approx 15.53 \text{ km}$$

$$c = \frac{b}{\sin B} (\sin C) = \frac{30}{\sin 30^\circ} (\sin 135^\circ) \approx 42.43 \text{ km}$$

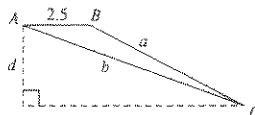


$$32. A = 20^\circ, B = 90^\circ + 63^\circ = 153^\circ, c = 10 \left(\frac{1}{4} \right) = 2.5$$

$$C = 180^\circ - 20^\circ - 153^\circ = 7^\circ$$

$$b = \frac{c}{\sin C} (\sin B) = \frac{2.5 \sin 153^\circ}{\sin 7^\circ} \approx 9.31$$

$$d \approx b \sin A \approx 9.31 \sin 20^\circ \approx 3.2 \text{ miles}$$



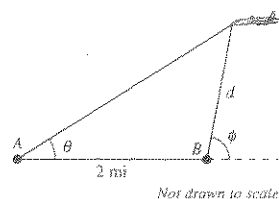
$$33. \frac{\sin(42^\circ - \theta)}{10} = \frac{\sin 48^\circ}{17}$$

$$\sin(42^\circ - \theta) = \frac{10}{17} \sin 48^\circ \approx 0.4371$$

$$42^\circ - \theta \approx 25.919$$

$$\theta \approx 16.1^\circ$$

34. (a)


 (b) Third angle in triangle = α

$$\theta + \alpha + (180^\circ - \phi) = 180^\circ \Rightarrow \alpha = \phi - \theta$$

$$\frac{d}{\sin \theta} = \frac{2}{\sin \alpha}$$

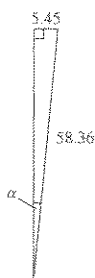
$$d = \frac{2 \sin \theta}{\sin \alpha} = \frac{2 \sin \theta}{\sin(\phi - \theta)}$$

$$35. (a) \sin \alpha = \frac{5.45}{58.36} \approx 0.0934$$

$$\alpha \approx 5.36^\circ$$

$$(b) \frac{d}{\sin \beta} = \frac{58.36}{\sin \theta} \Rightarrow \sin \beta = \frac{d \sin \theta}{58.36}$$

$$\beta = \sin^{-1} \left[\frac{d \sin \theta}{58.36} \right]$$



$$(c) \theta + \beta + 90^\circ + 5.36^\circ = 180^\circ \Rightarrow \beta = 84.64^\circ - \theta$$

$$d = \sin \beta \left(\frac{58.36}{\sin \theta} \right) = \sin(84.64^\circ - \theta) \frac{58.36}{\sin \theta}$$

θ	10°	20°	30°	40°	50°	60°
d	324.1	154.2	95.2	63.8	43.3	28.1

$$36. (a) \frac{\sin \alpha}{9} = \frac{\sin \beta}{18}$$

$$\sin \alpha = 0.5 \sin \beta$$

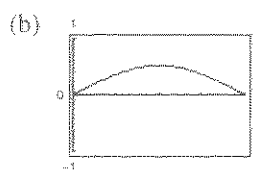
$$\alpha = \arcsin(0.5 \sin \beta)$$

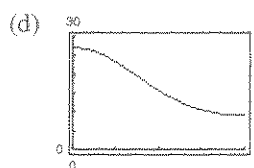
$$(c) \gamma = \pi - \alpha - \beta = \pi - \beta - \arcsin(0.5 \sin \beta)$$

$$\frac{c}{\sin \gamma} = \frac{18}{\sin \beta}$$

$$c = \frac{18 \sin \gamma}{\sin \beta}$$

$$= \frac{18 \sin[\pi - \beta - \arcsin(0.5 \sin \beta)]}{\sin \beta}$$


 Domain: $0 < \beta < \pi$

 Range: $0 < \alpha \leq \pi/6$

 Domain: $0 < \beta < \pi$

 Range: $9 < c < 27$

β	0.4	0.8	1.2	1.6	2.0	2.4	2.8
α	0.1960	0.3669	0.4848	0.5234	0.4720	0.3445	0.1683
c	25.95	23.07	19.19	15.33	12.29	10.31	9.27

 As $\beta \rightarrow 0$, $c \rightarrow 27$.

 As $\beta \rightarrow \pi$, $c \rightarrow 9$.

37. False. If just the three angles are known, the triangle cannot be solved.

 38. True. No angle could be 90° .

39. Yes, the Law of Sines can be used to solve a right triangle if you are given at least one side and one angle, or two sides.

41. Given: $A = 45^\circ$, $B = 52^\circ$, $a = 16$

$$C = 180^\circ - 45^\circ - 52^\circ = 83^\circ$$

$$b = \frac{a}{\sin A}(\sin B) = \frac{16}{\sin 45^\circ}(\sin 52^\circ) \approx 17.83$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{16}{\sin 45^\circ}(\sin 83^\circ) \approx 22.46$$

$$(a + b) \sin\left(\frac{C}{2}\right) = c \cos\left(\frac{A - B}{2}\right)$$

$$(16 + 17.83) \sin\left(\frac{83^\circ}{2}\right) = 22.46 \cos\left(\frac{45^\circ - 52^\circ}{2}\right)$$

$$22.42 = 22.42$$

43. $\tan \theta = \frac{\sin \theta}{\cos \theta} = -\frac{12}{5}$

$$\sec \theta = \frac{13}{5}$$

$$\cot \theta = -\frac{5}{12}$$

$$\csc \theta = -\frac{13}{12}$$

40. Answers will vary. $A = 36^\circ$, $a = 5$

(a) $b = 4$ one solution

(b) $b = 7$ two solutions [$h = b \sin A < a < b$]

(c) $b = 10$ no solution [$a < h = b \sin A$]

42. Given: $A = 42^\circ$, $B = 60^\circ$, $a = 24$

$$C = 180^\circ - 42^\circ - 60^\circ = 78^\circ$$

$$b = \frac{a}{\sin A}(\sin B) = \frac{24}{\sin 42^\circ}(\sin 60^\circ) \approx 31.06$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{24}{\sin 42^\circ}(\sin 78^\circ) \approx 35.08$$

$$(a - b) \cos\left(\frac{C}{2}\right) = c \sin\left(\frac{A - B}{2}\right)$$

$$(24 - 31.06) \cos\left(\frac{78^\circ}{2}\right) = 35.08 \sin\left(\frac{42^\circ - 60^\circ}{2}\right)$$

$$-5.49 = -5.49$$

44. $\cot \theta = \frac{9}{2}$

$$\sin \theta = -\frac{2}{\sqrt{85}} = -\frac{2\sqrt{85}}{85}$$

$$\cos \theta = \cot \theta \cdot \sin \theta$$

$$= \frac{9}{2} \left(-\frac{2}{\sqrt{85}} \right) = -\frac{9}{\sqrt{85}} = -\frac{9\sqrt{85}}{85}$$

$$\sec \theta = -\frac{\sqrt{85}}{9}$$

45. $6 \sin 8\theta \cos 3\theta = 6\left(\frac{1}{2}\right)[\sin(8\theta + 3\theta) + \sin(8\theta - 3\theta)] = 3(\sin 11\theta + \sin 5\theta)$

46. $2 \cos 2\theta \cos 5\theta = 2\left(\frac{1}{2}\right)[\cos(2\theta - 5\theta) + \cos(2\theta + 5\theta)] = \cos 3\theta + \cos 7\theta$

47. $3 \cos \frac{\pi}{6} \sin \frac{5\pi}{3} = 3\left(\frac{1}{2}\right)\left[\sin\left(\frac{\pi}{6} + \frac{5\pi}{3}\right) - \sin\left(\frac{\pi}{6} - \frac{5\pi}{3}\right)\right]$

$$= \frac{3}{2}\left[\sin\left(\frac{11\pi}{6}\right) - \sin\left(-\frac{3\pi}{2}\right)\right]$$

$$= \frac{3}{2}\left[-\frac{1}{2} - 1\right] = -\frac{9}{4}$$

48. $\frac{5}{2} \sin \frac{3\pi}{4} \sin \frac{5\pi}{6} = \frac{5}{2} \cdot \frac{1}{2}\left[\cos\left(\frac{3\pi}{4} - \frac{5\pi}{6}\right) - \cos\left(\frac{3\pi}{4} + \frac{5\pi}{6}\right)\right]$

$$= \frac{5}{4}\left[\cos\left(-\frac{\pi}{12}\right) - \cos\left(\frac{19\pi}{12}\right)\right]$$

$$= \frac{5}{4}\left[\cos\left(\frac{\pi}{12}\right) - \cos\left(\frac{19\pi}{12}\right)\right]$$

Section 6.2 Law of Cosines

■ If ABC is any oblique triangle with sides a , b , and c , then the Law of Cosines says:

$$(a) \quad a^2 = b^2 + c^2 - 2bc \cos A \quad \text{or} \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$(b) \quad b^2 = a^2 + c^2 - 2ac \cos B \quad \text{or} \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$(c) \quad c^2 = a^2 + b^2 - 2ab \cos C \quad \text{or} \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

■ You should be able to use the Law of Cosines to solve an oblique triangle for the remaining three parts, given:

(a) Three sides (SSS)

(b) Two sides and their included angle (SAS)

■ Given any triangle with sides of lengths a , b , and c , then the area of the triangle is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a+b+c}{2}. \quad (\text{Heron's Formula})$$

Vocabulary Check

1. $c^2 = a^2 + b^2 - 2ab \cos C$

2. Heron's Area

3. $\frac{1}{2}bh$, $\sqrt{s(s-a)(s-b)(s-c)}$

1. Given: $a = 12$, $b = 16$, $c = 18$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{16^2 + 18^2 - 12^2}{2(16)(18)} \approx 0.75694 \Rightarrow A \approx 40.80^\circ$$

$$\sin B = \frac{b \sin A}{a} \approx 0.8712 \Rightarrow B \approx 60.61^\circ$$

$$C \approx 180^\circ - 60.61^\circ - 40.80^\circ = 78.59^\circ$$

2. Given: $a = 8$, $b = 18$, $c = 12$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{18^2 + 12^2 - 8^2}{2(18)(12)} \approx 0.9352 \Rightarrow A \approx 20.74^\circ$$

$$\sin C = \frac{c \sin A}{a} \approx 0.5312 \Rightarrow C \approx 32.09^\circ$$

$$B \approx 180^\circ - 32.09^\circ - 20.74^\circ = 127.17^\circ$$

3. Given: $a = 8.5$, $b = 9.2$, $c = 10.8$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{9.2^2 + 10.8^2 - 8.5^2}{2(9.2)(10.8)} \approx 0.6493 \Rightarrow A \approx 49.51^\circ$$

$$\sin B = \frac{b \sin A}{a} \approx \frac{9.2 \sin 49.51^\circ}{8.5} \approx 0.82315 \Rightarrow B \approx 55.40^\circ$$

$$C \approx 180^\circ - 55.40^\circ - 49.51^\circ = 75.09^\circ$$

4. Given:
- $a = 4.2$
- ,
- $b = 5.4$
- ,
- $c = 2.1$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{4.2^2 + 5.4^2 - 2.1^2}{2(4.2)(5.4)} \approx 0.9345 \Rightarrow C \approx 20.85^\circ$$

$$\sin A = \frac{a \sin C}{c} \approx \frac{4.2 \sin 20.8^\circ}{2.1} \approx 0.7102 \Rightarrow A \approx 45.38^\circ$$

$$B \approx 180^\circ - 20.85^\circ - 45.38^\circ = 113.77^\circ$$

5. Given:
- $a = 10$
- ,
- $c = 15$
- ,
- $B = 20^\circ$

$$b^2 = a^2 + c^2 - 2ac \cos B = 100 + 225 - 2(10)(15) \cos 20^\circ \approx 43.0922 \Rightarrow b \approx 6.56 \text{ mm}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \approx \frac{43.0922 + 225 - 100}{2(6.56)(15)} \approx 0.8541 \Rightarrow A \approx 31.40^\circ$$

$$C \approx 180^\circ - 20^\circ - 31.40^\circ = 128.60^\circ$$

6. Given:
- $a = 16$
- ,
- $c = 10$
- ,
- $B = 40^\circ$

$$b^2 = a^2 + c^2 - 2ac \cos B = 256 + 100 - 2(16)(10) \cos 40^\circ \approx 110.8658 \Rightarrow b \approx 10.5293 \approx 10.53 \text{ km}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \approx \frac{256 + 110.8658 - 100}{2(16)(10.53)} \approx 0.7920 \Rightarrow C \approx 37.62^\circ$$

$$A \approx 180^\circ - 40^\circ - 37.62^\circ = 102.38^\circ$$

7. Given:
- $a = 10.4$
- ,
- $c = 12.5$
- ,
- $B = 50^\circ 30' = 50.5^\circ$

$$b^2 = a^2 + c^2 - 2ac \cos B = 10.4^2 + 12.5^2 - 2(10.4)(12.5) \cos 50.5^\circ \approx 99.0297 \Rightarrow b \approx 9.95 \text{ ft}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \approx \frac{99.0297 + 12.5^2 - 10.4^2}{2(9.95)(12.5)} \approx 0.5914 \Rightarrow A \approx 53.75^\circ = 53^\circ 45'$$

$$C \approx 180^\circ - 50.5^\circ - 53.75^\circ = 75.75^\circ = 75^\circ 45'$$

8. Given:
- $a = 20.2$
- ,
- $c = 6.2$
- ,
- $B = 80^\circ 45' = 80.75^\circ$

$$b^2 = a^2 + c^2 - 2ac \cos B = 20.2^2 + 6.2^2 - 2(20.2)(6.2) \cos 80.75^\circ \approx 406.2172 \Rightarrow b \approx 20.15 \text{ miles}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \approx \frac{20.2^2 + 406.2172 - 6.2^2}{2(20.2)(20.15)} \approx 0.9528 \Rightarrow C \approx 17.68^\circ \approx 17^\circ 41'$$

$$A \approx 180^\circ - 17.68^\circ - 80.75^\circ = 81.57^\circ = 81^\circ 34'$$

9. Given:
- $a = 6$
- ,
- $b = 8$
- ,
- $c = 12$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{64 + 144 - 36}{2(8)(12)} \approx 0.8958 \Rightarrow A \approx 26.4^\circ$$

$$\sin B = \frac{b \sin A}{a} \approx \frac{8 \sin 26.4^\circ}{6} \approx 0.5928 \Rightarrow B \approx 36.3^\circ$$

$$C \approx 180^\circ - 26.4^\circ - 36.3^\circ = 117.3^\circ$$

10. Given:
- $a = 9, b = 3, c = 11$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{3^2 + 11^2 - 9^2}{2(3)(11)} \approx 0.7424 \Rightarrow A \approx 42.1^\circ$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{9^2 + 3^2 - 11^2}{2(9)(3)} \approx -0.574 \Rightarrow C \approx 125.0^\circ$$

$$B = 180^\circ - A - C \approx 12.9^\circ$$

11. Given:
- $A = 50^\circ, b = 15, c = 30$

$$a^2 = b^2 + c^2 - 2bc \cos A = 225 + 900 - 2(15)(30) \cos 50^\circ \approx 546.49 \Rightarrow a \approx 23.38$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx \frac{546.49 + 900 - 225}{2(23.4)(30)} \approx 0.8708 \Rightarrow B \approx 29.4^\circ$$

$$C = 180^\circ - A - B \approx 180^\circ - 50^\circ - 29.5^\circ = 100.6^\circ$$

- 12.
- $C = 108^\circ, a = 10, b = 7$

$$c^2 = a^2 + b^2 - 2ab \cos C = 10^2 + 7^2 - 2(10)(7) \cos 108^\circ \approx 192.2624 \Rightarrow c \approx 13.9$$

$$\sin B = \frac{\sin C}{c} b = \frac{\sin 108^\circ}{13.9}(7) \approx 0.4789 \Rightarrow B \approx 28.7^\circ$$

$$A = 180^\circ - 108^\circ - 28.7^\circ = 43.3^\circ$$

13. Given:
- $a = 9, b = 12, c = 15$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{81 + 144 - 225}{2(9)(12)} = 0 \Rightarrow C = 90^\circ$$

$$\sin A = \frac{9}{15} = \frac{3}{5} \Rightarrow A \approx 36.9^\circ$$

$$B \approx 180^\circ - 90^\circ - 36.9^\circ = 53.1^\circ$$

14. Given:
- $a = 45, b = 30, c = 72$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{45^2 + 30^2 - 72^2}{2(45)(30)} \approx -0.8367 \Rightarrow C \approx 146.8^\circ$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{45^2 + 72^2 - 30^2}{2(45)(72)} \approx 0.9736 \Rightarrow B \approx 13.2^\circ$$

$$A = 180^\circ - B - C = 20.0^\circ$$

15. Given:
- $a = 75.4, b = 48, c = 48$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{48^2 + 48^2 - 75.4^2}{2(48)(48)} \approx -0.2338 \Rightarrow A \approx 103.5^\circ$$

$$\sin B = \frac{b \sin A}{a} \approx \frac{48 \sin(103.5^\circ)}{75.4} \approx 0.6190 \Rightarrow B \approx 38.2^\circ$$

$$C = B \approx 38.2^\circ \text{ (Because of roundoff error, } A + B + C \neq 180^\circ \text{.)}$$

16. Given: $a = 1.42, b = 0.75, c = 1.25$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(0.75)^2 + (1.25)^2 - (1.42)^2}{2(0.75)(1.25)} = 0.05792 \Rightarrow A \approx 86.7^\circ$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{(1.42)^2 + (1.25)^2 - (0.75)^2}{2(1.42)(1.25)} \approx 0.8497 \Rightarrow B \approx 31.8^\circ$$

$$180^\circ - 86.7^\circ - 31.8^\circ \approx 61.5^\circ$$

17. Given: $B = 8^\circ 15' = 8.25^\circ, a = 26, c = 18$

$$b^2 = a^2 + c^2 - 2ac \cos B = 26^2 + 18^2 - 2(26)(18) \cos(8.25^\circ) \approx 73.6863 \Rightarrow b \approx 8.58$$

$$\sin C = \frac{c \sin B}{b} \approx \frac{18 \sin(8.25^\circ)}{8.58} \approx 0.3 \Rightarrow C \approx 17.51^\circ \approx 17^\circ 31'$$

$$A = 180^\circ - B - C \approx 180^\circ - 8.25^\circ - 17.51^\circ = 154.24^\circ \approx 154^\circ 14'$$

18. Given: $B = 10^\circ 35' \approx 10.583^\circ, a = 40, c = 30$

$$b^2 = a^2 + c^2 - 2ac \cos B \approx 140.8268 \Rightarrow b \approx 11.87$$

$$\sin A = \frac{a \sin B}{b} \approx 0.6189 \Rightarrow A \approx 141.75^\circ \approx 141^\circ 45'$$

$$C = 180^\circ - A - B = 27.67^\circ \text{ or } 27^\circ 40'$$

19. Given: $B = 75^\circ 20' \approx 75.33^\circ, a = 6.2, c = 9.5$

$$\begin{aligned} b^2 &= a^2 + c^2 - 2ac \cos B \\ &= 6.2^2 + 9.5^2 - 2(6.2)(9.5) \cos(75.33^\circ) \\ &= 98.86 \end{aligned}$$

$$b \approx 9.94$$

$$\sin C = \frac{c \sin B}{b} = \frac{9.5 \sin(75.33^\circ)}{9.94}$$

$$\approx 0.9246 \Rightarrow C \approx 67.6^\circ$$

$$A = 180^\circ - B - C \approx 37.1^\circ$$

20. Given: $C = 15^\circ 15' \approx 15.25^\circ, a = 6.25, b = 2.15$

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab \cos C \\ &= 6.25^2 + 2.15^2 - 2(6.25)(2.15) \cos 15.25^\circ \\ &\approx 17.7563 \Rightarrow c \approx 4.21 \end{aligned}$$

$$\sin B = \frac{b \sin C}{c}$$

$$\approx 0.1342 \Rightarrow B \approx 7.71^\circ \approx 7^\circ 43'$$

$$A = 180^\circ - B - C \approx 157.04^\circ \approx 157^\circ 2'$$

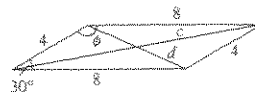
21. $d^2 = 4^2 + 8^2 - 2(4)(8) \cos 30^\circ$

$$\approx 24.57 \Rightarrow d \approx 4.96$$

$$2\phi = 360^\circ - 2\theta \Rightarrow \phi = 150^\circ$$

$$c^2 = 4^2 + 8^2 - 2(4)(8) \cos 150^\circ \approx 135.43$$

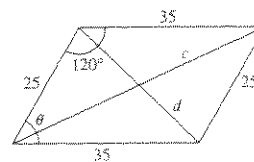
$$c \approx 11.64$$



22. $c^2 = 25^2 + 35^2 - 2(25)(35) \cos 120^\circ = 2725 \Rightarrow c \approx 52.2$

$$2\theta = 360^\circ - 2(120^\circ) = 120^\circ \Rightarrow \theta = 60^\circ$$

$$d^2 = 25^2 + 35^2 - 2(25)(35) \cos 60^\circ = 975 \Rightarrow d \approx 31.22$$



$$23. \cos \phi = \frac{10^2 + 14^2 - 20^2}{2(10)(14)}$$

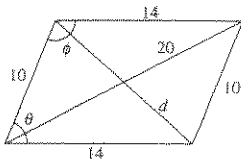
$$\phi \approx 111.8^\circ$$

$$2\theta \approx 360^\circ - 2(111.80^\circ)$$

$$\theta \approx 68.2^\circ$$

$$d^2 = 10^2 + 14^2 - 2(10)(14) \cos 68.2^\circ$$

$$d \approx 13.86$$

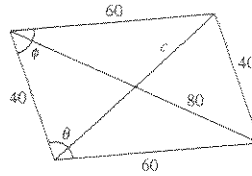


$$24. \cos \theta = \frac{40^2 + 60^2 - 80^2}{2(40)(60)} \approx -\frac{1}{4} \Rightarrow \theta \approx 104.5^\circ$$

$$2\phi \approx 360^\circ - 2(104.5^\circ) = 151^\circ \Rightarrow \phi \approx 75.5^\circ$$

$$c^2 \approx 40^2 + 60^2 - 2(40)(60) \cos 75.5^\circ = 4000$$

$$c \approx 63.25$$



$$25. \cos \alpha = \frac{15^2 + 12.5^2 - 10^2}{2(15)(12.5)} = 0.75 \Rightarrow \alpha \approx 41.41^\circ$$

$$\cos \beta = \frac{15^2 + 10^2 - 12.5^2}{2(15)(10)} = 0.5625 \Rightarrow \beta \approx 55.77^\circ$$

$$\delta = 180^\circ - 41.41^\circ - 55.77^\circ \approx 82.82^\circ$$

$$\mu = 180^\circ - \delta \approx 97.18^\circ$$

$$b^2 = 12.5^2 + 10^2 - 2(12.5)(10) \cos(97.18^\circ) \approx 287.50$$

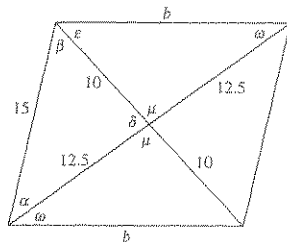
$$b \approx 16.96$$

$$\sin \omega = \frac{10}{16.96} \sin \mu \approx 0.585 \Rightarrow \omega \approx 35.8^\circ$$

$$\sin \epsilon = \frac{12.5}{16.96} \sin \mu \approx 0.731 \Rightarrow \epsilon \approx 47^\circ$$

$$\theta = \alpha + \omega \approx 77.2^\circ$$

$$\phi = \beta + \epsilon \approx 102.8^\circ$$



$$26. \cos \alpha = \frac{25^2 + 17.5^2 - 25^2}{2(25)(17.5)}$$

$$\alpha \approx 69.513^\circ$$

$$\beta \approx 180^\circ - \alpha \approx 110.487^\circ$$

$$a^2 = 17.5^2 + 25^2 - 2(17.5)(25) \cos 110.487^\circ$$

$$a \approx 35.18$$

$$z = 180^\circ - 2\alpha \approx 40.974^\circ$$

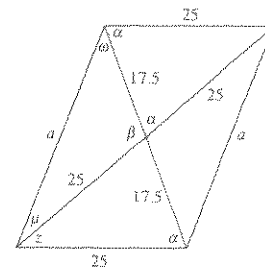
$$\cos \mu = \frac{25^2 + 35.18^2 - 17.5^2}{2(25)(35.18)}$$

$$\mu \approx 27.772^\circ$$

$$\theta = \mu + z \approx 68.7^\circ$$

$$\omega = 180^\circ - \mu - \beta \approx 41.741^\circ$$

$$\phi = \omega + \alpha \approx 111.3^\circ$$



$$27. \text{ Given: } a = 12, b = 24, c = 18$$

$$s = \frac{a + b + c}{2} = 27$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{27(15)(3)(9)}$$

$$\approx 104.57 \text{ square inches}$$

$$28. \text{ Given: } a = 25, b = 35, c = 32$$

$$s = \frac{a + b + c}{2} = 46$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{46(21)(11)(14)}$$

$$\approx 385.70 \text{ square meters}$$

29. Given:
- $a = 5, b = 8, c = 10$

$$s = \frac{a + b + c}{2} = \frac{23}{2} = 11.5$$

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{11.5(6.5)(3.5)(1.5)} \\ &\approx 19.81 \text{ square units}\end{aligned}$$

31. Given:
- $a = 1.24, b = 2.45, c = 1.25$

$$s = \frac{a + b + c}{2} = 2.47$$

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{2.47(1.23)(0.02)(1.22)} \\ &\approx 0.27 \text{ square feet}\end{aligned}$$

33. Given:
- $a = 3.5, b = 10.2, c = 9$

$$s = \frac{a + b + c}{2} = 11.35$$

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{11.35(7.85)(1.15)(2.35)} \\ &\approx 15.52 \text{ square units}\end{aligned}$$

35. Given:
- $a = 10.59, b = 6.65, c = 12.31$

$$s = \frac{a + b + c}{2} = 14.775$$

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{14.775(4.185)(8.125)(2.465)} \\ &\approx 35.19 \text{ square units}\end{aligned}$$

- 37.
- $B = 105^\circ + 32^\circ = 137^\circ$

$$\begin{aligned}b^2 &= a^2 + c^2 - 2ac \cdot \cos B \\ &= 648^2 + 810^2 - 2(648)(810) \cos(137^\circ) \\ &= 1,843,749.862 \\ b &= 1357.8 \text{ miles}\end{aligned}$$

From the Law of Sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \sin A = \frac{a}{b} \sin B = \frac{648}{1357.8} \sin(137^\circ) \approx 0.32548$$

$$\Rightarrow A \approx 19^\circ \Rightarrow \text{Bearing S } 56^\circ \text{ W (or } 236^\circ \text{ for airplane navigation)}$$

- 30.
- $s = \frac{a + b + c}{2} = \frac{14 + 17 + 7}{2} = 19$

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{19(5)(2)(12)} \\ &\approx 47.7 \text{ square units}\end{aligned}$$

32. Given:
- $a = 2.4, b = 2.75, c = 2.25$

$$s = \frac{a + b + c}{2} = 3.7$$

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{3.7(1.3)(0.95)(1.45)} \\ &\approx 2.57 \text{ square centimeters}\end{aligned}$$

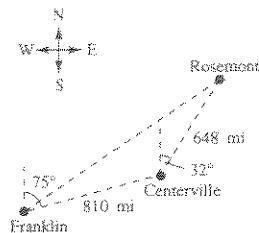
34. Given:
- $a = 75.4, b = 52, c = 52$

$$s = \frac{75.4 + 52 + 52}{2} = 89.7$$

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{89.7(14.3)(37.7)(37.7)} \\ &\approx 1350 \text{ square units}\end{aligned}$$

- 36.
- $s = \frac{a + b + c}{2} = \frac{4.45 + 1.85 + 3.00}{2} = 4.65$

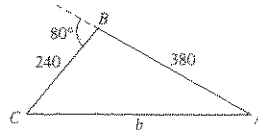
$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{4.65(0.2)(2.8)(1.65)} \\ &\approx 2.07 \text{ square units}\end{aligned}$$



38. Angle at $B = 180^\circ - 80^\circ = 100^\circ$

$$b^2 = 240^2 + 380^2 - 2(240)(380) \cos 100^\circ$$

$$\approx 233,673.4 \Rightarrow b \approx 483.4 \text{ meters}$$



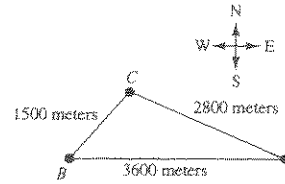
39. $\cos B = \frac{1500^2 + 3600^2 - 2800^2}{2(1500)(3600)} \approx 0.6824 \Rightarrow B \approx 46.97^\circ$

Bearing at B : $90^\circ - 46.97^\circ$: N 43.03° E

$$\cos C = \frac{1500^2 + 2800^2 - 3600^2}{2(1500)(2800)} \approx -0.3417 \Rightarrow C \approx 109.98^\circ$$

Bearing at C : $C - (90^\circ - 46.97^\circ) = 66.95^\circ$

S 66.95° E



40. $\cos \theta = \frac{2^2 + 3^2 - (4.5)^2}{2(2)(3)} \approx -0.60417$

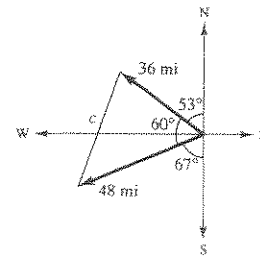
$$\theta \approx 127.2^\circ$$

41. $C = 180^\circ - 53^\circ - 67^\circ = 60^\circ$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$= 36^2 + 48^2 - 2(36)(48)(0.5) = 1872$$

$$c \approx 43.3 \text{ mi}$$



42. The angles at the base of the tower are 96° and 84° . The longer guy wire g_1 is given by:

$$g_1^2 = 75^2 + 100^2 - 2(75)(100) \cos 96^\circ \approx 17,192.9 \Rightarrow g_1 \approx 131.1 \text{ feet}$$

The shorter guy wire g_2 is given by: $g_2^2 = 75^2 + 100^2 - 2(75)(100) \cos 84^\circ \approx 14,057.1 \Rightarrow g_2 \approx 118.6 \text{ feet}$

43. $\overline{RS} = \sqrt{8^2 + 10^2} = \sqrt{164} = 2\sqrt{41} \approx 12.8 \text{ feet}$

$$\overline{PQ} = \frac{1}{2} \sqrt{16^2 + 10^2} = \frac{1}{2} \sqrt{356} = \sqrt{89} \approx 9.4 \text{ feet}$$

$$\tan P = \frac{10}{16}$$

$$P = \arctan \frac{5}{8} \approx 32.0^\circ$$

$$\overline{QS} = \sqrt{8^2 + 9.4^2 - 2(8)(9.4) \cos 32^\circ}$$

$$\approx \sqrt{24.81} \approx 5.0 \text{ feet}$$

45. $s = \frac{a + b + c}{2} = \frac{145 + 257 + 290}{2} = 346$

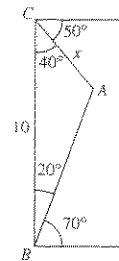
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{346(201)(89)(56)} \approx 18,617.7 \text{ square feet}$$

44. $A = 180^\circ - 40^\circ - 20^\circ = 120^\circ$

$$x = \frac{(\sin 20^\circ)}{\sin 120^\circ}(10)$$

$$\approx 3.95 \text{ feet}$$



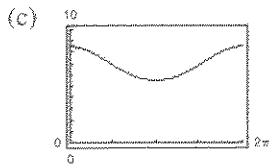
46. The height is $h = 70 \sin 70^\circ \approx 65.778$.

Area = base \times height

$$= (100)(65.778) \approx 6577.8 \text{ square meters}$$

47. (a) $7^2 = 1.5^2 + x^2 - 2(1.5)(x) \cos \theta$

$$49 = 2.25 + x^2 - 3x \cos \theta$$



(d) Note that $x = 8.5$ when $\theta = 0$ and $\theta = 2\pi$, and $x = 5.5$ when $\theta = \pi$.

Thus, the distance is

$$2(8.5 - 5.5) = 2(3) = 6 \text{ inches.}$$

(b) $x^2 - 3x \cos \theta = 46.75$

$$x^2 - 3x \cos \theta + \left(\frac{3 \cos \theta}{2}\right)^2 = 46.75 + \left(\frac{3 \cos \theta}{2}\right)^2$$

$$\left[x - \frac{3 \cos \theta}{2}\right]^2 = \frac{187}{4} + \frac{9 \cos^2 \theta}{4}$$

$$x - \frac{3 \cos \theta}{2} = \pm \sqrt{\frac{187 + 9 \cos^2 \theta}{4}}$$

Choosing the positive values of x , we have

$$x = \frac{1}{2}(3 \cos \theta + \sqrt{9 \cos^2 \theta + 187}).$$

48. (a) $d^2 = 10^2 + 7^2 - 2(10)(7) \cos \theta \Rightarrow d = \sqrt{149 - 140 \cos \theta}$

(b) $\theta = \arccos\left[\frac{10^2 + 7^2 - d^2}{2(10)(7)}\right] = \arccos\left[\frac{149 - d^2}{140}\right]$

(c) $s = \frac{360^\circ - \theta}{360^\circ}(2\pi r) = \frac{(360^\circ - \theta)\pi}{45^\circ}$

(d)

d (inches)	9	10	12	13	14	15	16
θ (degrees)	60.9°	69.5°	88.0°	98.2°	109.6°	122.9°	139.8°
s (inches)	20.88	20.28	18.99	18.28	17.48	16.55	15.37

49. False. This is not a triangle! $5 + 10 < 16$

50. True. The third side is found by the Law of Cosines. The other angles are determined by the Law of Sines.

51. False. $s = \frac{a + b + c}{2}$, not $\frac{a + b + c}{3}$.

$$\begin{aligned} 52. \frac{1}{2}bc(1 + \cos A) &= \frac{1}{2}bc\left[1 + \frac{b^2 + c^2 - a^2}{2bc}\right] \\ &= \frac{1}{2}bc\left[\frac{2bc + b^2 + c^2 - a^2}{2bc}\right] \\ &= \frac{1}{4}[(b + c)^2 - a^2] \\ &= \frac{1}{4}[(b + c) + a][(b + c) - a] \\ &= \frac{b + c + a}{2} \cdot \frac{b + c - a}{2} \\ &= \frac{a + b + c}{2} \cdot \frac{-a + b + c}{2} \end{aligned}$$

$$\begin{aligned} 53. \frac{1}{2}bc(1 - \cos A) &= \frac{1}{2}bc\left[1 - \frac{b^2 + c^2 - a^2}{2bc}\right] \\ &= \frac{1}{2}bc\left[\frac{2bc - b^2 - c^2 + a^2}{2bc}\right] \\ &= \frac{1}{4}[a^2 - (b - c)^2] \\ &= \frac{1}{4}[a - b + c](a + b - c) \\ &= \left(\frac{a - b + c}{2}\right)\left(\frac{a + b - c}{2}\right) \end{aligned}$$

$$\begin{aligned} 54. \frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} &= \frac{b^2 + c^2 - a^2}{a(2bc)} + \frac{a^2 + c^2 - b^2}{b(2ac)} + \frac{a^2 + b^2 - c^2}{c(2ab)} \\ &= \frac{a^2 + b^2 + c^2}{2abc} \end{aligned}$$

55. Since $0 < C < 180^\circ$, $\cos\left(\frac{C}{2}\right) = \sqrt{\frac{1 + \cos C}{2}}$.

Hence, $\cos\left(\frac{C}{2}\right) = \sqrt{\frac{1 + (a^2 + b^2 - c^2)/(2ab)}{2}} = \sqrt{\frac{2ab + a^2 + b^2 - c^2}{4ab}}$.

On the other hand,

$$\begin{aligned} s(s - c) &= \frac{1}{2}(a + b + c)\left(\frac{1}{2}(a + b + c) - c\right) \\ &= \frac{1}{2}(a + b + c)\frac{1}{2}(a + b - c) \\ &= \frac{1}{4}((a + b)^2 - c^2) \\ &= \frac{1}{4}(a^2 + b^2 + 2ab - c^2). \end{aligned}$$

Thus, $\sqrt{\frac{s(s - c)}{ab}} = \sqrt{\frac{a^2 + b^2 + 2ab - c^2}{4ab}}$ and we have verified that $\cos\left(\frac{C}{2}\right) = \sqrt{\frac{s(s - c)}{ab}}$.

56. Since $0 < C < 180^\circ$, $\sin\left(\frac{C}{2}\right) = \sqrt{\frac{1 - \cos C}{2}}$.

Hence, $\sin\left(\frac{C}{2}\right) = \sqrt{\frac{1 - (a^2 + b^2 - c^2)/(2ab)}{2}} = \sqrt{\frac{2ab - a^2 - b^2 + c^2}{4ab}}$.

On the other hand,

$$\begin{aligned} (s - a)(s - b) &= \left[\frac{1}{2}(a + b + c) - a\right]\left[\frac{1}{2}(a + b + c) - b\right] \\ &= \frac{1}{2}(b + c - a)\frac{1}{2}(a + c - b) \\ &= \frac{1}{4}[c - (a - b)][c + (a - b)] \\ &= \frac{1}{4}[c^2 - (a - b)^2] \\ &= \frac{1}{4}(c^2 - a^2 - b^2 + 2ab). \end{aligned}$$

Thus, $\sqrt{\frac{(s - a)(s - b)}{ab}} = \sqrt{\frac{c^2 - a^2 - b^2 + 2ab}{4ab}} = \sin\left(\frac{C}{2}\right)$.

57. Given: $a = 12$, $b = 30$, $A = 20^\circ$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$12^2 = 30^2 + c^2 - 2(30)(c) \cos 20^\circ$$

$$c^2 - (60 \cos 20^\circ)c + 756 = 0$$

Solving this quadratic equation, $c \approx 21.97, 34.41$.

—CONTINUED—

57. —CONTINUED—

For $c = 21.97$,

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx \frac{12^2 + 21.97^2 - 30^2}{2(12)(21.97)} \approx -0.5184 \Rightarrow B \approx 121.2^\circ$$

$$C \approx 180^\circ - 121.2^\circ - 20^\circ = 38.8^\circ$$

For $c = 34.41$,

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx \frac{12^2 + 34.41^2 - 30^2}{2(12)(34.41)} \approx 0.5183 \Rightarrow B \approx 58.8^\circ$$

$$C \approx 180^\circ - 58.8^\circ - 20^\circ = 101.2^\circ$$

Using the Law of Sines, $\sin B = \frac{b \sin A}{a} = \frac{30 \sin 20^\circ}{12} \approx 0.8551 \Rightarrow B \approx 58.8^\circ$ or 121.2° .For $B = 58.8^\circ$, $C = 180^\circ - 58.8^\circ - 20^\circ = 101.2^\circ$ and $c = \frac{a \sin C}{\sin A} \approx 34.42$.For $B = 121.2^\circ$, $C = 180^\circ - 121.2^\circ - 20^\circ = 38.8^\circ$ and $c = \frac{a \sin C}{\sin A} \approx 21.98$.

58. Answers will vary. The Law of Cosines can be used to solve the single-solution case of SSA. It cannot be used for the no-solution case.

59. $\arcsin(-1) = -\frac{\pi}{2}$ because $\sin\left(-\frac{\pi}{2}\right) = -1$.

60. $\cos^{-1} 0 = \frac{\pi}{2}$ because $\cos \frac{\pi}{2} = 0$.

61. $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$ because $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$.

62. $\arcsin\left(\frac{-\sqrt{3}}{2}\right) = \frac{-\pi}{3}$ because $\sin\left(\frac{-\pi}{3}\right) = \frac{-\sqrt{3}}{2}$.

Section 6.3 Vectors in the Plane

- A vector \mathbf{v} is the collection of all directed line segments that are equivalent to a given directed line segment \overrightarrow{PQ} .
- You should be able to *geometrically* perform the operations of vector addition and scalar multiplication.
- The component form of the vector with initial point $P = (p_1, p_2)$ and terminal point $Q = (q_1, q_2)$ is

$$\overrightarrow{PQ} = \langle q_1 - p_1, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle = \mathbf{v}.$$
- The magnitude of $\mathbf{v} = \langle v_1, v_2 \rangle$ is given by $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$.
- You should be able to perform the operations of scalar multiplication and vector addition in component form.
- You should know the following properties of vector addition and scalar multiplication.

(a) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$	(b) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
(c) $\mathbf{u} + \mathbf{0} = \mathbf{u}$	(d) $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
(e) $c(d\mathbf{u}) = (cd)\mathbf{u}$	(f) $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
(g) $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$	(h) $1(\mathbf{u}) = \mathbf{u}, 0\mathbf{u} = \mathbf{0}$
(i) $\ c\mathbf{v}\ = c \ \mathbf{v}\ $	

—CONTINUED—

Section 6.3 —CONTINUED—

- A unit vector in the direction of \mathbf{v} is given by $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$.
- The standard unit vectors are $\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$. $\mathbf{v} = \langle v_1, v_2 \rangle$ can be written as $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j}$.
- A vector \mathbf{v} with magnitude $\|\mathbf{v}\|$ and direction θ can be written as $\mathbf{v} = a\mathbf{i} + b\mathbf{j} = \|\mathbf{v}\|(\cos \theta)\mathbf{i} + \|\mathbf{v}\|(\sin \theta)\mathbf{j}$ where $\tan \theta = b/a$.

Vocabulary Check

- | | | |
|-----------------------------|----------------------|---|
| 1. directed line segment | 2. initial, terminal | 3. magnitude |
| 4. vector | 5. standard position | 6. unit vector |
| 7. multiplication, addition | 8. resultant | 9. linear combination, horizontal, vertical |

1. $\mathbf{u} = \langle 6 - 2, 5 - 4 \rangle = \langle 4, 1 \rangle = \mathbf{v}$

2. $\mathbf{u} = \langle -3 - 0, -4 - 4 \rangle = \langle -3, -8 \rangle$

$\mathbf{v} = \langle 0 - 3, -5 - 3 \rangle = \langle -3, -8 \rangle$

$\mathbf{u} = \mathbf{v}$

3. Initial point: $(0, 0)$

Terminal point: $(4, 3)$

$\mathbf{v} = \langle 4 - 0, 3 - 0 \rangle = \langle 4, 3 \rangle$

$\|\mathbf{v}\| = \sqrt{4^2 + 3^2} = 5$

4. Initial point: $(0, 0)$

Terminal point: $(4, -2)$

$\mathbf{v} = \langle 4 - 0, -2 - 0 \rangle = \langle 4, -2 \rangle$

$\|\mathbf{v}\| = \sqrt{4^2 + (-2)^2} = \sqrt{20} = 2\sqrt{5} \approx 4.47$

5. Initial point: $(2, 2)$

Terminal point: $(-1, 4)$

$\mathbf{v} = \langle -1 - 2, 4 - 2 \rangle = \langle -3, 2 \rangle$

$\|\mathbf{v}\| = \sqrt{(-3)^2 + 2^2} = \sqrt{13} \approx 3.61$

6. Initial point: $(-1, -1)$

Terminal point: $(3, 5)$

$\mathbf{v} = \langle 3 - (-1), 5 - (-1) \rangle = \langle 4, 6 \rangle$

$\|\mathbf{v}\| = \sqrt{4^2 + 6^2} = \sqrt{52} = 2\sqrt{13} \approx 7.21$

7. Initial point: $(3, -2)$

Terminal point: $(3, 3)$

$\mathbf{v} = \langle 3 - 3, 3 - (-2) \rangle = \langle 0, 5 \rangle$

$\|\mathbf{v}\| = 5$

8. Initial point: $(-4, -1)$

Terminal point: $(3, -1)$

$\mathbf{v} = \langle 3 - (-4), -1 - (-1) \rangle = \langle 7, 0 \rangle$

$\|\mathbf{v}\| = \sqrt{7^2 + 0^2} = 7$

9. Initial point: $(\frac{2}{5}, 1)$

Terminal point: $(1, \frac{2}{5})$

$\mathbf{v} = \langle 1 - \frac{2}{5}, \frac{2}{5} - 1 \rangle = \langle \frac{3}{5}, -\frac{3}{5} \rangle$

$\|\mathbf{v}\| = \sqrt{(\frac{3}{5})^2 + (-\frac{3}{5})^2} = \sqrt{\frac{18}{25}} = \frac{3}{5}\sqrt{2}$

10. Initial point: $(\frac{7}{2}, 0)$

Terminal point: $(0, -\frac{7}{2})$

$\mathbf{v} = \langle 0 - \frac{7}{2}, -\frac{7}{2} - 0 \rangle = \langle -\frac{7}{2}, -\frac{7}{2} \rangle$

$\|\mathbf{v}\| = \sqrt{(-\frac{7}{2})^2 + (-\frac{7}{2})^2} = \sqrt{\frac{98}{4}} = \frac{7}{2}\sqrt{2}$

11. Initial point: $\left(-\frac{2}{3}, -1\right)$

Terminal point: $\left(\frac{1}{2}, \frac{4}{5}\right)$

$$\mathbf{v} = \left\langle \frac{1}{2} - \left(-\frac{2}{3}\right), \frac{4}{5} - (-1) \right\rangle = \left\langle \frac{7}{6}, \frac{9}{5} \right\rangle$$

$$\|\mathbf{v}\| = \sqrt{\left(\frac{7}{6}\right)^2 + \left(\frac{9}{5}\right)^2} = \frac{\sqrt{4141}}{30} \approx 2.1450$$

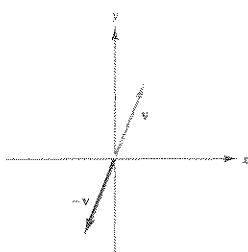
12. Initial point: $\left(\frac{5}{2}, -2\right)$

Terminal point: $\left(1, \frac{2}{5}\right)$

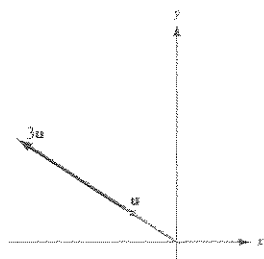
$$\mathbf{v} = \left\langle 1 - \frac{5}{2}, \frac{2}{5} - (-2) \right\rangle = \left\langle -\frac{3}{2}, \frac{12}{5} \right\rangle$$

$$\|\mathbf{v}\| = \sqrt{\left(-\frac{3}{2}\right)^2 + \left(\frac{12}{5}\right)^2} = \frac{3\sqrt{89}}{10} \approx 2.8302$$

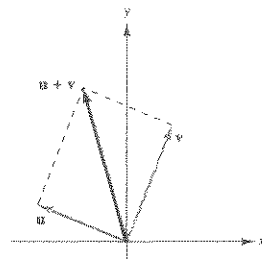
13. $-\mathbf{v}$



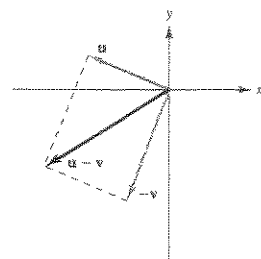
14. $3\mathbf{u}$



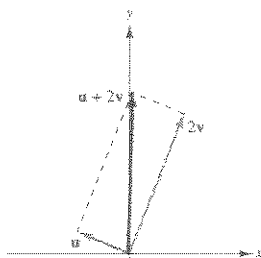
15. $\mathbf{u} + \mathbf{v}$



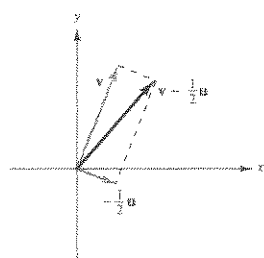
16. $\mathbf{u} - \mathbf{v}$



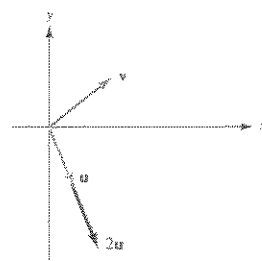
17. $\mathbf{u} + 2\mathbf{v}$



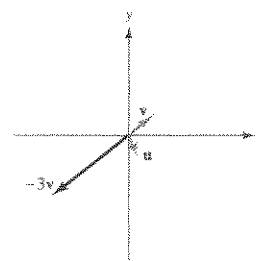
18. $\mathbf{v} - \frac{1}{2}\mathbf{u}$



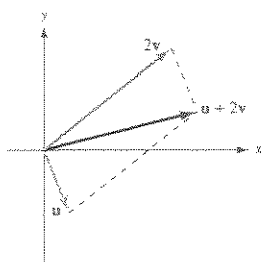
19.



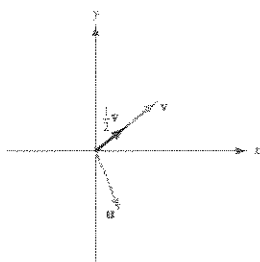
20.



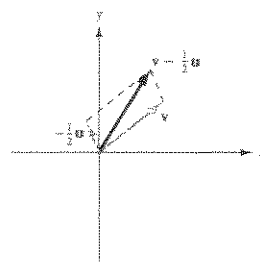
21.



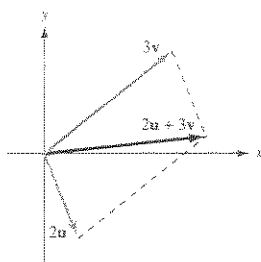
22.



23.



24.



25. $\mathbf{u} = \langle 4, 2 \rangle, \mathbf{v} = \langle 7, 1 \rangle$

(a) $\mathbf{u} + \mathbf{v} = \langle 11, 3 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -3, 1 \rangle$

(c) $2\mathbf{u} - 3\mathbf{v} = \langle 8, 4 \rangle - \langle 21, 3 \rangle = \langle -13, 1 \rangle$

(d) $\mathbf{v} + 4\mathbf{u} = \langle 7, 1 \rangle + \langle 16, 8 \rangle = \langle 23, 9 \rangle$

26. (a) $\mathbf{u} + \mathbf{v} = \langle 5, 3 \rangle + \langle -4, 0 \rangle = \langle 1, 3 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle 5, 3 \rangle - \langle -4, 0 \rangle = \langle 9, 3 \rangle$

(c) $2\mathbf{u} - 3\mathbf{v} = 2\langle 5, 3 \rangle - 3\langle -4, 0 \rangle = \langle 22, 6 \rangle$

(d) $\mathbf{v} + 4\mathbf{u} = \langle -4, 0 \rangle + 4\langle 5, 3 \rangle = \langle 16, 12 \rangle$

27. $\mathbf{u} = \langle -6, -8 \rangle, \mathbf{v} = \langle 2, 4 \rangle$

(a) $\mathbf{u} + \mathbf{v} = \langle -4, -4 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -8, -12 \rangle$

(c) $2\mathbf{u} - 3\mathbf{v} = \langle -12, -16 \rangle - \langle 6, 12 \rangle = \langle -18, -28 \rangle$

(d) $\mathbf{v} + 4\mathbf{u} = \langle 2, 4 \rangle + 4\langle -6, -8 \rangle = \langle -22, -28 \rangle$

28. (a) $\mathbf{u} + \mathbf{v} = \langle 0, -5 \rangle + \langle -3, 9 \rangle = \langle -3, 4 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle 3, -14 \rangle$

(c) $2\mathbf{u} - 3\mathbf{v} = \langle 0, -10 \rangle - \langle -9, 27 \rangle = \langle 9, -37 \rangle$

(d) $\mathbf{v} + 4\mathbf{u} = \langle -3, 9 \rangle + \langle 0, -20 \rangle = \langle -3, -11 \rangle$

29. $\mathbf{u} = \mathbf{i} + \mathbf{j}, \mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$

(a) $\mathbf{u} + \mathbf{v} = 3\mathbf{i} - 2\mathbf{j}$

(b) $\mathbf{u} - \mathbf{v} = -\mathbf{i} + 4\mathbf{j}$

(c) $2\mathbf{u} - 3\mathbf{v} = (2\mathbf{i} + 2\mathbf{j}) - (6\mathbf{i} - 9\mathbf{j}) = -4\mathbf{i} + 11\mathbf{j}$

(d) $\mathbf{v} + 4\mathbf{u} = (2\mathbf{i} - 3\mathbf{j}) + (4\mathbf{i} + 4\mathbf{j}) = 6\mathbf{i} + \mathbf{j}$

30. $\mathbf{u} = 2\mathbf{i} - \mathbf{j}, \mathbf{v} = -\mathbf{i} + \mathbf{j}$

(a) $\mathbf{u} + \mathbf{v} = \mathbf{i}$

(b) $\mathbf{u} - \mathbf{v} = 3\mathbf{i} - 2\mathbf{j}$

(c) $2\mathbf{u} - 3\mathbf{v} = (4\mathbf{i} - 2\mathbf{j}) - (-3\mathbf{i} + 3\mathbf{j}) = 7\mathbf{i} - 5\mathbf{j}$

(d) $\mathbf{v} + 4\mathbf{u} = 7\mathbf{i} - 3\mathbf{j}$

31. $\mathbf{w} = \mathbf{u} + \mathbf{v}$

32. $\mathbf{v} = \mathbf{w} - \mathbf{u}$

33. $\mathbf{u} = \mathbf{w} - \mathbf{v}$

34. $2\mathbf{v} = 2(\mathbf{w} - \mathbf{u})$
 $= 2\mathbf{w} - 2\mathbf{u}$

35. $\|\langle 6, 0 \rangle\| = 6$

Unit vector: $\frac{1}{6}\langle 6, 0 \rangle = \langle 1, 0 \rangle$

36. $\mathbf{u} = \langle 0, -2 \rangle$

$\|\mathbf{u}\| = 2$

Unit vector:

$\frac{1}{2}\langle 0, -2 \rangle = \langle 0, -1 \rangle$

37. $\mathbf{v} = \langle -1, 1 \rangle$

$\|\mathbf{v}\| = \sqrt{2}$

Unit vector = $\frac{1}{\|\mathbf{v}\|}\mathbf{v}$

$= \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$

$= \left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$

38. $\mathbf{v} = \langle 3, -4 \rangle$

$\|\mathbf{v}\| = \sqrt{3^2 + (-4)^2} = 5$

Unit vector: $\frac{1}{5}\langle 3, -4 \rangle = \left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle$

39. $\|\mathbf{v}\| = \|\langle -24, -7 \rangle\| = \sqrt{(-24)^2 + (-7)^2} = 25$

Unit vector: $\frac{1}{25}\langle -24, -7 \rangle = \left\langle -\frac{24}{25}, -\frac{7}{25} \right\rangle$

40. $\mathbf{v} = \langle 8, -20 \rangle$

$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{64 + 400}}\langle 8, -20 \rangle = \frac{1}{\sqrt{29}}\langle 2, -5 \rangle = \left\langle \frac{2}{\sqrt{29}}, -\frac{5}{\sqrt{29}} \right\rangle = \left\langle \frac{2\sqrt{29}}{29}, -\frac{5\sqrt{29}}{29} \right\rangle$

41. $\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{16 + 9}}(4\mathbf{i} - 3\mathbf{j}) = \frac{1}{5}(4\mathbf{i} - 3\mathbf{j}) = \frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}$

42. $\mathbf{w} = \mathbf{i} - 2\mathbf{j}$

$$\mathbf{u} = \frac{1}{\|\mathbf{w}\|} \mathbf{w} = \frac{1}{\sqrt{1^2 + (-2)^2}} (\mathbf{i} - 2\mathbf{j}) = \frac{1}{\sqrt{5}} (\mathbf{i} - 2\mathbf{j}) = \frac{\sqrt{5}}{5} \mathbf{i} - \frac{2\sqrt{5}}{5} \mathbf{j}$$

43. $\mathbf{u} = \frac{1}{2}(2\mathbf{j}) = \mathbf{j}$

44. $\mathbf{w} = -3\mathbf{i}$

$\|\mathbf{w}\| = 3$

$\mathbf{u} = \frac{1}{3}(-3\mathbf{i}) = -\mathbf{i}$

$$\begin{aligned} 45. \quad 8\left(\frac{1}{\|\mathbf{u}\|} \mathbf{u}\right) &= 8\left(\frac{1}{\sqrt{5^2 + 6^2}} \langle 5, 6 \rangle\right) \\ &= \frac{8}{\sqrt{61}} \langle 5, 6 \rangle \\ &= \left\langle \frac{40\sqrt{61}}{61}, \frac{48\sqrt{61}}{61} \right\rangle \end{aligned}$$

$$\begin{aligned} 46. \quad \mathbf{v} &= 3\left(\frac{1}{\|\mathbf{u}\|} \mathbf{u}\right) \\ &= 3\left(\frac{1}{\sqrt{4^2 + (-4)^2}} \langle 4, -4 \rangle\right) \\ &= 3\left(\frac{1}{4\sqrt{2}} \langle 4, -4 \rangle\right) \\ &= \left\langle \frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}} \right\rangle \\ &= \left\langle \frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2} \right\rangle \end{aligned}$$

$$\begin{aligned} 47. \quad 7\left(\frac{1}{\|\mathbf{u}\|} \mathbf{u}\right) &= 7\left(\frac{1}{\sqrt{3^2 + 4^2}} \langle 3, 4 \rangle\right) \\ &= \frac{7}{5} \langle 3, 4 \rangle \\ &= \left\langle \frac{21}{5}, \frac{28}{5} \right\rangle \\ &= \frac{21}{5} \mathbf{i} + \frac{28}{5} \mathbf{j} \end{aligned}$$

$$\begin{aligned} 48. \quad \mathbf{v} &= 10\left(\frac{1}{\|\mathbf{u}\|} \mathbf{u}\right) \\ &= 10\left(\frac{1}{\sqrt{4 + 9}} \langle 2, -3 \rangle\right) \\ &= \left\langle \frac{20}{\sqrt{13}}, -\frac{30}{\sqrt{13}} \right\rangle \\ &= \frac{20}{\sqrt{13}} \mathbf{i} - \frac{30}{\sqrt{13}} \mathbf{j} \\ &= \frac{20\sqrt{13}}{13} \mathbf{i} - \frac{30\sqrt{13}}{13} \mathbf{j} \end{aligned}$$

$$\begin{aligned} 49. \quad 8\left(\frac{1}{\|\mathbf{u}\|} \mathbf{u}\right) &= 8\left(\frac{1}{2} \langle -2, 0 \rangle\right) \\ &= 4 \langle -2, 0 \rangle \\ &= \langle -8, 0 \rangle \\ &= -8\mathbf{i} \end{aligned}$$

$$\begin{aligned} 50. \quad \mathbf{v} &= 4\left(\frac{1}{\|\mathbf{u}\|} \mathbf{u}\right) \\ &= 4\left(\frac{1}{5} \langle 0, 5 \rangle\right) \\ &= \langle 0, 4 \rangle = 4\mathbf{j} \end{aligned}$$

51. $\mathbf{v} = \langle 4 - (-3), 5 - 1 \rangle = \langle 7, 4 \rangle = 7\mathbf{i} + 4\mathbf{j}$

52. $\langle 3 - 0, 6 - (-2) \rangle = \langle 3, 8 \rangle = 3\mathbf{i} + 8\mathbf{j}$

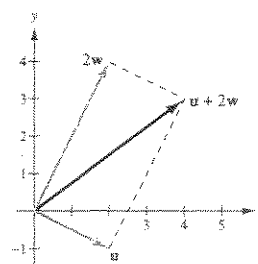
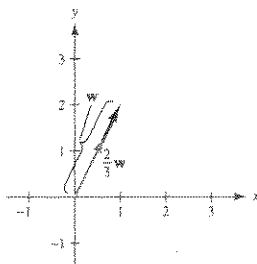
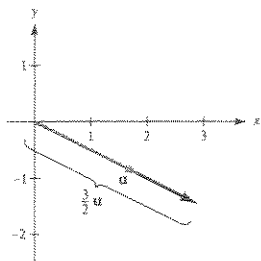
53. $\mathbf{v} = \langle 2 - (-1), 3 - (-5) \rangle = \langle 3, 8 \rangle = 3\mathbf{i} + 8\mathbf{j}$

54. $\langle 0 - (-6), 1 - 4 \rangle = \langle 6, -3 \rangle = 6\mathbf{i} - 3\mathbf{j}$

$$\begin{aligned} 55. \quad \mathbf{v} &= \frac{3}{2} \mathbf{u} \\ &= \frac{3}{2} (2\mathbf{i} - \mathbf{j}) = 3\mathbf{i} - \frac{3}{2} \mathbf{j} \\ &= \left\langle 3, -\frac{3}{2} \right\rangle \end{aligned}$$

$$\begin{aligned} 56. \quad \mathbf{v} &= \frac{2}{3} \mathbf{w} \\ &= \frac{2}{3} \langle 1, 2 \rangle \\ &= \left\langle \frac{2}{3}, \frac{4}{3} \right\rangle \end{aligned}$$

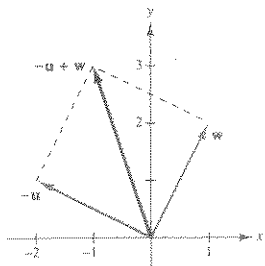
$$\begin{aligned} 57. \quad \mathbf{v} &= \mathbf{u} + 2\mathbf{w} \\ &= (2\mathbf{i} - \mathbf{j}) + 2(\mathbf{i} + 2\mathbf{j}) \\ &= 4\mathbf{i} + 3\mathbf{j} = \langle 4, 3 \rangle \end{aligned}$$



58. $\mathbf{v} = -\mathbf{u} + \mathbf{w}$

$$= -(2\mathbf{i} - \mathbf{j}) + (\mathbf{i} + 2\mathbf{j})$$

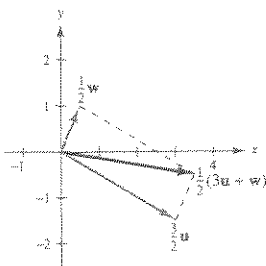
$$= -\mathbf{i} + 3\mathbf{j} = \langle -1, 3 \rangle$$



59. $\mathbf{v} = \frac{1}{2}(3\mathbf{u} + \mathbf{w})$

$$= \frac{1}{2}(3\langle 2, -1 \rangle + \langle 1, 2 \rangle)$$

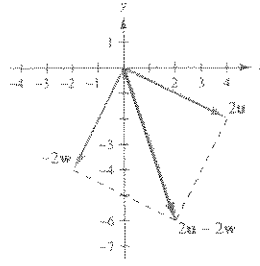
$$= \left\langle \frac{7}{2}, -\frac{1}{2} \right\rangle$$



60. $\mathbf{v} = 2\mathbf{u} - 2\mathbf{w}$

$$= \langle 4, -2 \rangle - \langle 2, 4 \rangle$$

$$= \langle 2, -6 \rangle$$



61. $\mathbf{v} = 5(\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{j})$

$$\|\mathbf{v}\| = 5, \theta = 30^\circ$$

62. $\mathbf{v} = 8(\cos 135^\circ \mathbf{i} + \sin 135^\circ \mathbf{j})$

$$\|\mathbf{v}\| = 8, \theta = 135^\circ$$

63. $\mathbf{v} = 6\mathbf{i} - 6\mathbf{j}$

$$\|\mathbf{v}\| = \sqrt{6^2 + (-6)^2} = \sqrt{72} = 6\sqrt{2}$$

$$\tan \theta = -\frac{6}{6} = -1$$

Since \mathbf{v} lies in Quadrant IV, $\theta = 315^\circ$.

64. $\mathbf{v} = -4\mathbf{i} - 7\mathbf{j}$, Quadrant III

$$\|\mathbf{v}\| = \sqrt{(-4)^2 + (-7)^2} = \sqrt{65}$$

$$\tan \theta = \frac{-7}{-4} = \frac{7}{4} \Rightarrow \theta \approx 240.3^\circ$$

65. $\mathbf{v} = -2\mathbf{i} + 5\mathbf{j}$

$$\|\mathbf{v}\| = \sqrt{(-2)^2 + 5^2} = \sqrt{29}$$

$$\tan \theta = -\frac{5}{2}$$

Since \mathbf{v} lies in Quadrant II, $\theta \approx 111.8^\circ$.

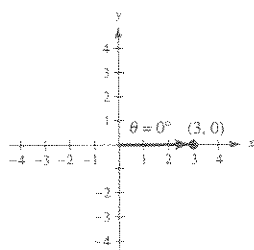
66. $\mathbf{v} = 12\mathbf{i} + 15\mathbf{j}$, Quadrant I

$$\|\mathbf{v}\| = \sqrt{12^2 + 15^2} = \sqrt{369} = 3\sqrt{41}$$

$$\tan \theta = \frac{15}{12} \Rightarrow \theta \approx 51.3^\circ$$

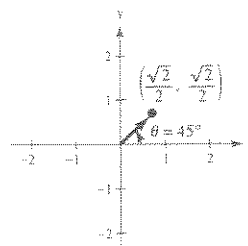
67. $\mathbf{v} = \langle 3 \cos 0^\circ, 3 \sin 0^\circ \rangle$

$$= \langle 3, 0 \rangle$$



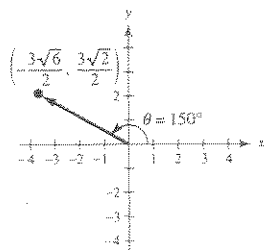
68. $\mathbf{v} = \langle \cos 45^\circ, \sin 45^\circ \rangle$

$$= \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$$

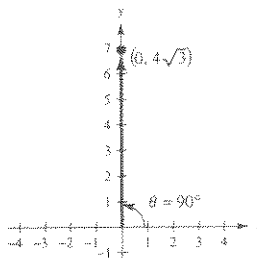


69. $\mathbf{v} = \langle 3\sqrt{2} \cos 150^\circ, 3\sqrt{2} \sin 150^\circ \rangle$

$$= \left\langle -\frac{3\sqrt{6}}{2}, \frac{3\sqrt{2}}{2} \right\rangle$$

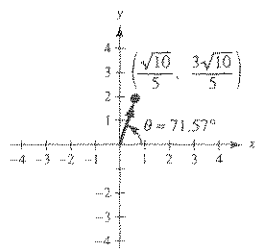


70. $\mathbf{v} = 4\sqrt{3} \langle \cos 90^\circ, \sin 90^\circ \rangle = \langle 0, 4\sqrt{3} \rangle$

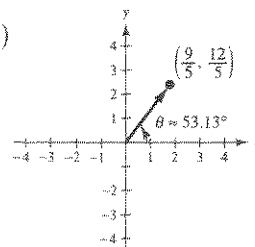


$$\begin{aligned}
 71. \quad \mathbf{v} &= 2\left(\frac{1}{\sqrt{3^2 + 1^2}}\right)(\mathbf{i} + 3\mathbf{j}) \\
 &= \frac{2}{\sqrt{10}}(\mathbf{i} + 3\mathbf{j}) \\
 &= \frac{\sqrt{10}}{5}\mathbf{i} + \frac{3\sqrt{10}}{5}\mathbf{j} \\
 &= \left\langle \frac{\sqrt{10}}{5}, \frac{3\sqrt{10}}{5} \right\rangle
 \end{aligned}$$

$$\tan \theta = \frac{3}{1} \Rightarrow \theta \approx 71.57^\circ$$



$$\begin{aligned}
 72. \quad \mathbf{v} &= 3\left(\frac{1}{\sqrt{3^2 + 4^2}}\right)(3\mathbf{i} + 4\mathbf{j}) \\
 &= \frac{3}{5}(3\mathbf{i} + 4\mathbf{j}) \\
 &= \frac{9}{5}\mathbf{i} + \frac{12}{5}\mathbf{j} = \left\langle \frac{9}{5}, \frac{12}{5} \right\rangle
 \end{aligned}$$



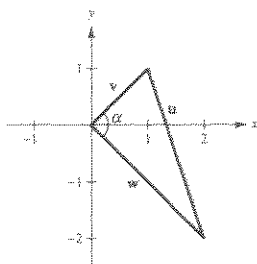
$$\begin{aligned}
 73. \quad \mathbf{u} &= \langle 5 \cos 60^\circ, 5 \sin 60^\circ \rangle = \left\langle \frac{5}{2}, \frac{5\sqrt{3}}{2} \right\rangle \\
 \mathbf{v} &= \langle 5 \cos 90^\circ, 5 \sin 90^\circ \rangle = \langle 0, 5 \rangle \\
 \mathbf{u} + \mathbf{v} &= \left\langle \frac{5}{2}, \frac{5\sqrt{3}}{2} \right\rangle + \langle 0, 5 \rangle = \left\langle \frac{5}{2}, 5 + \frac{5\sqrt{3}}{2} \right\rangle
 \end{aligned}$$

$$\begin{aligned}
 74. \quad \mathbf{u} &= \langle 2 \cos 30^\circ, 2 \sin 30^\circ \rangle = \langle \sqrt{3}, 1 \rangle \\
 \mathbf{v} &= \langle 2 \cos 90^\circ, 2 \sin 90^\circ \rangle = \langle 0, 2 \rangle \\
 \mathbf{u} + \mathbf{v} &= \langle \sqrt{3}, 3 \rangle
 \end{aligned}$$

$$\begin{aligned}
 75. \quad \mathbf{u} &= \langle 20 \cos 45^\circ, 20 \sin 45^\circ \rangle = \langle 10\sqrt{2}, 10\sqrt{2} \rangle \\
 \mathbf{v} &= \langle 50 \cos 150^\circ, 50 \sin 150^\circ \rangle = \langle -25\sqrt{3}, 25 \rangle \\
 \mathbf{u} + \mathbf{v} &= \langle 10\sqrt{2} - 25\sqrt{3}, 10\sqrt{2} + 25 \rangle
 \end{aligned}$$

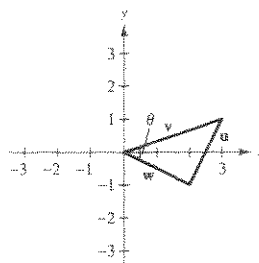
$$\begin{aligned}
 76. \quad \mathbf{u} &= \langle 35 \cos 25^\circ, 35 \sin 25^\circ \rangle = \langle 31.72, 14.79 \rangle \\
 \mathbf{v} &= \langle 50 \cos 120^\circ, 50 \sin 120^\circ \rangle = \langle -25, 25\sqrt{3} \rangle \\
 \mathbf{u} + \mathbf{v} &\approx \langle 6.72, 58.09 \rangle
 \end{aligned}$$

$$\begin{aligned}
 77. \quad \mathbf{v} &= \mathbf{i} + \mathbf{j} \\
 \mathbf{w} &= 2(\mathbf{i} - \mathbf{j}) \\
 \mathbf{u} &= \mathbf{v} - \mathbf{w} = -\mathbf{i} + 3\mathbf{j} \\
 \|\mathbf{v}\| &= \sqrt{2} \\
 \|\mathbf{w}\| &= 2\sqrt{2} \\
 \|\mathbf{v} - \mathbf{w}\| &= \sqrt{10}
 \end{aligned}$$



$$\begin{aligned}
 \cos \alpha &= \frac{\|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - \|\mathbf{v} - \mathbf{w}\|^2}{2\|\mathbf{v}\|\|\mathbf{w}\|} = \frac{2 + 8 - 10}{2\sqrt{2} \cdot 2\sqrt{2}} = 0 \\
 \alpha &= 90^\circ
 \end{aligned}$$

$$\begin{aligned}
 78. \quad \mathbf{v} &= 3\mathbf{i} + \mathbf{j} \\
 \mathbf{w} &= 2\mathbf{i} - \mathbf{j} \\
 \mathbf{u} &= \mathbf{v} - \mathbf{w} = \mathbf{i} + 2\mathbf{j} \\
 \cos \theta &= \frac{\|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - \|\mathbf{v} - \mathbf{w}\|^2}{2\|\mathbf{v}\|\|\mathbf{w}\|} = \frac{10 + 5 - 5}{2\sqrt{10}\sqrt{5}} = \frac{\sqrt{2}}{2} \\
 \theta &= 45^\circ
 \end{aligned}$$



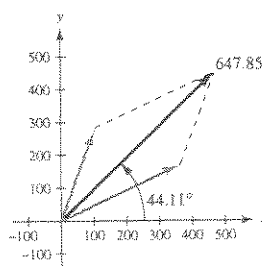
79. $\mathbf{u} = 400 \cos 25^\circ \mathbf{i} + 400 \sin 25^\circ \mathbf{j}$

$\mathbf{v} = 300 \cos 70^\circ \mathbf{i} + 300 \sin 70^\circ \mathbf{j}$

$\mathbf{u} + \mathbf{v} \approx 465.13\mathbf{i} + 450.96\mathbf{j}$

$\|\mathbf{u} + \mathbf{v}\| \approx \sqrt{(465.13)^2 + (450.96)^2} \approx 647.85$

$\alpha = \arctan\left(\frac{450.96}{465.13}\right) \approx 44.11^\circ$



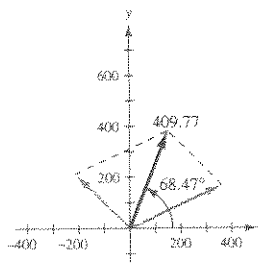
80. Analytically: $\mathbf{v} = 400(\cos 25^\circ, \sin 25^\circ)$

$\mathbf{u} = 300(\cos 135^\circ, \sin 135^\circ)$

$\mathbf{u} + \mathbf{v} = \langle 150.39, 381.18 \rangle$

$\|\mathbf{u} + \mathbf{v}\| = 409.77$

$\theta = \arctan\left(\frac{381.18}{150.39}\right) = 68.47^\circ$



81. Force One: $\mathbf{u} = 45\mathbf{i}$

Force Two: $\mathbf{v} = 60 \cos \theta \mathbf{i} + 60 \sin \theta \mathbf{j}$

Resultant Force: $\mathbf{u} + \mathbf{v} = (45 + 60 \cos \theta)\mathbf{i} + 60 \sin \theta \mathbf{j}$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{(45 + 60 \cos \theta)^2 + (60 \sin \theta)^2} = 90$

$2025 + 5400 \cos \theta + 3600 = 8100$

$5400 \cos \theta = 2475$

$\cos \theta = \frac{2475}{5400} \approx 0.4583$

$\theta \approx 62.7^\circ$

82. Force One: $\mathbf{u} = 3000\mathbf{i}$

Force Two: $\mathbf{v} = 1000 \cos \theta \mathbf{i} + 1000 \sin \theta \mathbf{j}$

Resultant Force: $\mathbf{u} + \mathbf{v} = (3000 + 1000 \cos \theta)\mathbf{i} + 1000 \sin \theta \mathbf{j}$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{(3000 + 1000 \cos \theta)^2 + (1000 \sin \theta)^2} = 3750$

$9,000,000 + 6,000,000 \cos \theta + 1,000,000 = 14,062,500$

$6,000,000 \cos \theta = 4,062,500$

$\cos \theta = \frac{4,062,500}{6,000,000} \approx 0.6771$

$\theta \approx 47.4^\circ$

83. Horizontal component of velocity: $70 \cos 40^\circ \approx 53.62$ ft/sec

Vertical component of velocity: $70 \sin 40^\circ \approx 45.0$ ft/sec

84. Horizontal component of velocity: $1200 \cos 4^\circ \approx 1197.1$ ft/sec

Vertical component of velocity: $1200 \sin 4^\circ \approx 83.7$ ft/sec

85. Rope \overrightarrow{AC} : $\mathbf{u} = 10\mathbf{i} - 24\mathbf{j}$

The vector lies in Quadrant IV and its reference angle is $\arctan(\frac{12}{5})$.

$$\mathbf{u} = \|\mathbf{u}\| [\cos(\arctan \frac{12}{5})\mathbf{i} - \sin(\arctan \frac{12}{5})\mathbf{j}]$$

Rope \overrightarrow{BC} : $\mathbf{v} = -20\mathbf{i} - 24\mathbf{j}$

The vector lies in Quadrant III and its reference angle is $\arctan(\frac{6}{5})$.

$$\mathbf{v} = \|\mathbf{v}\| [-\cos(\arctan \frac{6}{5})\mathbf{i} - \sin(\arctan \frac{6}{5})\mathbf{j}]$$

Resultant: $\mathbf{u} + \mathbf{v} = -5000\mathbf{j}$

$$\|\mathbf{u}\| \cos(\arctan \frac{12}{5}) - \|\mathbf{v}\| \cos(\arctan \frac{6}{5}) = 0$$

$$-\|\mathbf{u}\| \sin(\arctan \frac{12}{5}) - \|\mathbf{v}\| \sin(\arctan \frac{6}{5}) = -5000$$

Solving this system of equations yields:

$$T_{AC} = \|\mathbf{u}\| \approx 3611.1 \text{ pounds}$$

$$T_{BC} = \|\mathbf{v}\| \approx 2169.5 \text{ pounds}$$

86. Left cable: $\mathbf{u} = \|\mathbf{u}\|(\cos 155.7^\circ\mathbf{i} + \sin 155.7^\circ\mathbf{j})$

Right cable: $\mathbf{v} = \|\mathbf{v}\|(\cos 44.5^\circ\mathbf{i} + \sin 44.5^\circ\mathbf{j})$

$$\mathbf{u} + \mathbf{v} = 20,240\mathbf{j} \Rightarrow \|\mathbf{u}\| \cos 155.7^\circ + \|\mathbf{v}\| \cos 44.5^\circ = 0$$

$$\|\mathbf{u}\| \sin 155.7^\circ + \|\mathbf{v}\| \sin 44.5^\circ = 20,240$$

$$\Rightarrow -0.9114\|\mathbf{u}\| + 0.7133\|\mathbf{v}\| = 0$$

$$0.4115\|\mathbf{u}\| + 0.7009\|\mathbf{v}\| = 20,240$$

Solving this system for $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$ yields:

Tension of left cable: $\|\mathbf{u}\| = 15,485$ pounds

Tension of right cable: $\|\mathbf{v}\| = 19,786$ pounds

87. (a) Tow line 1: $\mathbf{u} = \|\mathbf{u}\|(\cos \theta\mathbf{i} + \sin \theta\mathbf{j})$

Tow line 2: $\mathbf{v} = \|\mathbf{u}\|(\cos(-\theta)\mathbf{i} + \sin(-\theta)\mathbf{j})$

Resultant: $\mathbf{u} + \mathbf{v} = 6000\mathbf{i} = [\|\mathbf{u}\| \cos \theta + \|\mathbf{u}\| \cos(-\theta)]\mathbf{i} \Rightarrow$

$$6000 = 2\|\mathbf{u}\| \cos \theta \Rightarrow \|\mathbf{u}\| \approx 3000 \sec \theta$$

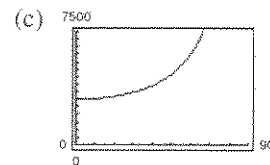
$$T = \|\mathbf{u}\| = 3000 \sec \theta$$

Domain: $0^\circ \leq \theta < 90^\circ$

(b)

θ	10°	20°	30°	40°	50°	60°
T	3046.3	3192.5	3464.1	3916.2	4667.2	6000.0

(d) The tension increases because the component in the direction of the motion of the barge decreases.



88. (a) $\mathbf{u} = \|\mathbf{u}\|(\cos(90^\circ - \theta)\mathbf{i} + \sin(90^\circ - \theta)\mathbf{j})$

$$\mathbf{v} = \|\mathbf{v}\|(\cos(90^\circ + \theta)\mathbf{i} + \sin(90^\circ + \theta)\mathbf{j})$$

$$\mathbf{u} + \mathbf{v} = 100\mathbf{j} \Rightarrow \|\mathbf{u}\|\cos(90^\circ - \theta) + \|\mathbf{v}\|\cos(90^\circ + \theta) = 0$$

$$\Rightarrow \|\mathbf{u}\| = \|\mathbf{v}\|, \text{ and}$$

$$100 = \|\mathbf{u}\|\sin(90^\circ - \theta) + \|\mathbf{u}\|\sin(90^\circ + \theta)$$

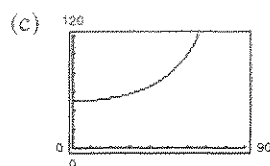
$$= 2\|\mathbf{u}\|\cos\theta$$

$$\text{Hence, } \|\mathbf{u}\| = T = \frac{50}{\cos\theta} = 50 \sec\theta.$$

$$\text{Domain: } 0^\circ \leq \theta < 90^\circ$$

(b)

θ	10°	20°	30°	40°	50°	60°
T	50.8	53.2	57.7	65.3	77.8	100



(d) The vertical component of the vectors decreases as θ increases.

89. Airspeed: $\mathbf{v} = 860(\cos 302^\circ\mathbf{i} + \sin 302^\circ\mathbf{j})$

Groundspeed: $\mathbf{u} = 800(\cos 310^\circ\mathbf{i} + \sin 310^\circ\mathbf{j})$

$$\mathbf{w} + \mathbf{v} = \mathbf{u}$$

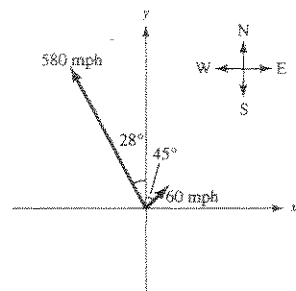
$$\mathbf{w} = \mathbf{u} - \mathbf{v} = 800(\cos 310^\circ\mathbf{i} + \sin 310^\circ\mathbf{j}) - 860(\cos 302^\circ\mathbf{i} + \sin 302^\circ\mathbf{j}) \approx 58.50\mathbf{i} + 116.49\mathbf{j}$$

$$\|\mathbf{w}\| = \sqrt{58.50^2 + 116.49^2} \approx 130.35 \text{ km/hr}$$

$$\theta = \arctan\left(\frac{116.49}{58.50}\right) \approx 63.3^\circ$$

Direction: N 26.7° E

90. (a)



(b) $\mathbf{w} = 60\langle \cos 45^\circ, \sin 45^\circ \rangle = \langle 30\sqrt{2}, 30\sqrt{2} \rangle$

(c) $\mathbf{v} = 580\langle \cos 118^\circ, \sin 118^\circ \rangle = \langle -272.3, 512.1 \rangle$

(d) $\mathbf{w} + \mathbf{v} = \langle -229.9, 554.5 \rangle$

$$\|\mathbf{w} + \mathbf{v}\| \approx 600.3 \text{ mph}$$

(e) $\tan\theta = \frac{554.5}{-229.9} \Rightarrow \theta \approx 112.5^\circ$

Direction: N 22.5° W (or 337.5° using airplane navigation)

91. (a) $\mathbf{u} = 220\mathbf{i}$, $\mathbf{v} = 150 \cos 30^\circ \mathbf{i} + 150 \sin 30^\circ \mathbf{j}$

$$\mathbf{u} + \mathbf{v} = (220 + 75\sqrt{3})\mathbf{i} + 75\mathbf{j}$$

$$\|\mathbf{u} + \mathbf{v}\| = \sqrt{(220 + 75\sqrt{3})^2 + 75^2} \approx 357.85 \text{ newtons}$$

$$\tan \theta = \frac{75}{220 + 75\sqrt{3}} \Rightarrow \theta \approx 12.1^\circ$$

(b) $\mathbf{u} + \mathbf{v} = 220\mathbf{i} + (150 \cos \theta \mathbf{i} + 150 \sin \theta \mathbf{j})$

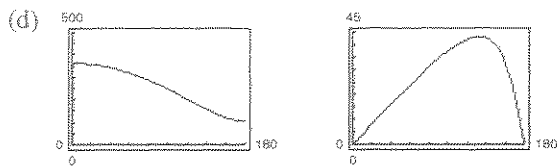
$$M = \|\mathbf{u} + \mathbf{v}\| = \sqrt{220^2 + 150^2(\cos^2 \theta + \sin^2 \theta) + 2(220)(150) \cos \theta}$$

$$= \sqrt{70,900 + 66,000 \cos \theta} = 10\sqrt{709 + 660 \cos \theta}$$

$$\alpha = \arctan\left(\frac{15 \sin \theta}{22 + 15 \cos \theta}\right)$$

 (c)

θ	0°	30°	60°	90°	120°	150°	180°
M	370.0	357.9	322.3	266.3	194.7	117.2	70.0
α	0°	12.1°	23.8°	34.3°	41.9°	39.8°	0°


 (e) For increasing θ , the two vectors tend to work against each other resulting in a decrease in the magnitude of the resultant.

92. (a) $\mathbf{T} = \|\mathbf{T}\|(\cos(90^\circ + \theta)\mathbf{i} + \sin(90^\circ + \theta)\mathbf{j})$

$$\mathbf{u} + \mathbf{w} + \mathbf{T} = \mathbf{0} \Rightarrow \|\mathbf{u}\| + \|\mathbf{T}\| \cos(90^\circ + \theta) = 0$$

$$\|\mathbf{u}\| - \|\mathbf{T}\| \sin \theta = 0$$

$$-1 + \|\mathbf{T}\| \sin(90^\circ + \theta) = 0$$

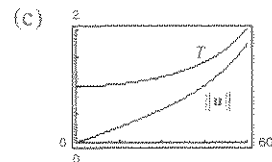
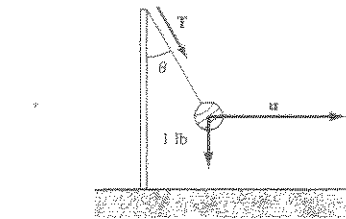
$$-1 + \|\mathbf{T}\| \cos \theta = 0 \Rightarrow$$

$$\|\mathbf{T}\| = \sec \theta, 0 \leq \theta < \frac{\pi}{2}$$

 Hence, $\|\mathbf{u}\| = \|\mathbf{T}\| \sin \theta = \sec \theta \sin \theta = \tan \theta, 0 \leq \theta < \pi/2$.

 (b)

θ	0°	10°	20°	30°	40°	50°	60°
$\ \mathbf{T}\ $	1	1.02	1.06	1.15	1.31	1.56	2
$\ \mathbf{u}\ $	0	0.18	0.36	0.58	0.84	1.19	1.73


 (d) Both $\|\mathbf{T}\|$ and $\|\mathbf{u}\|$ increase as θ increases, and approach each other in magnitude.

93. True. See page 424.

94. True, $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$.

 95. True. In fact, $a = b = 0$.

96. True

 97. True. \mathbf{a} and \mathbf{d} are parallel, and pointing in opposite directions.

 98. True. \mathbf{c} and \mathbf{s} are parallel, and pointing in the same direction.

99. True

100. False. In fact, $\mathbf{v} + \mathbf{s} = \mathbf{w}$.101. True. $\mathbf{a} + \mathbf{w} = 2\mathbf{a} = -2\mathbf{d}$ 102. True. $\mathbf{a} + \mathbf{d} = \mathbf{a} + (-\mathbf{a}) = \mathbf{0}$ 103. False. $\mathbf{u} - \mathbf{v} = 2\mathbf{u}$ and

$$-2(\mathbf{b} + \mathbf{t}) = -2(-2\mathbf{u}) = 4\mathbf{u}$$

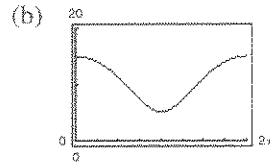
104. True

$$\mathbf{t} - \mathbf{w} = -\mathbf{s} \text{ and } \mathbf{b} - \mathbf{a} = -\mathbf{c} = -\mathbf{s}$$

105. (a) The angle between them is 0° .(b) The angle between them is 180° .(c) No. At most it can be equal to the sum when the angle between them is 0° .106. $\mathbf{F}_1 = \langle 10, 0 \rangle$, $\mathbf{F}_2 = 5\langle \cos \theta, \sin \theta \rangle$

$$(a) \quad \mathbf{F}_1 + \mathbf{F}_2 = \langle 10 + 5 \cos \theta, 5 \sin \theta \rangle$$

$$\begin{aligned} \|\mathbf{F}_1 + \mathbf{F}_2\| &= \sqrt{(10 + 5 \cos \theta)^2 + (5 \sin \theta)^2} \\ &= \sqrt{100 + 100 \cos \theta + 25 \cos^2 \theta + 25 \sin^2 \theta} \\ &= 5\sqrt{4 + 4 \cos \theta + \cos^2 \theta + \sin^2 \theta} \\ &= 5\sqrt{4 + 4 \cos \theta + 1} \\ &= 5\sqrt{5 + 4 \cos \theta} \end{aligned}$$

(c) Range: $[5, 15]$ Maximum is 15 when $\theta = 0$.Minimum is 5 when $\theta = \pi$.(d) The magnitude of the resultant is never 0 because the magnitudes of \mathbf{F}_1 and \mathbf{F}_2 are not the same.107. Let $\mathbf{v} = (\cos \theta)\mathbf{i} + (\sin \theta)\mathbf{j}$.

$$\|\mathbf{v}\| = \sqrt{\cos^2 \theta + \sin^2 \theta} = \sqrt{1} = 1$$

Therefore, \mathbf{v} is a unit vector for any value of θ .

108. The following program is written for a TI-82 or TI-83 graphing calculator. The program sketches two vectors $\mathbf{u} = a\mathbf{i} + b\mathbf{j}$ and $\mathbf{v} = c\mathbf{i} + d\mathbf{j}$ in standard position, and then sketches the vector difference $\mathbf{u} - \mathbf{v}$ using the parallelogram law.

PROGRAM: SUBVECT

:Input "ENTER A", A

:Input "ENTER B", B

:Input "ENTER C", C

:Input "ENTER D", D

:Line (0, 0, A, B)

:Line (0, 0, C, D)

:Pause

:A-C→E

:B-D→F

:Line (A, B, C, D)

:Line (A, B, E, F)

:Line (0, 0, E, F)

:Pause

:ClrDraw

:Stop

109. $\mathbf{u} = \langle 5 - 1, 2 - 6 \rangle = \langle 4, -4 \rangle$

$$\mathbf{v} = \langle 9 - 4, 4 - 5 \rangle = \langle 5, -1 \rangle$$

$$\mathbf{u} - \mathbf{v} = \langle -1, -3 \rangle$$

$$\mathbf{v} - \mathbf{u} = \langle 1, 3 \rangle$$

110. $\mathbf{u} = \langle 80 - 10, 80 - 60 \rangle = \langle 70, 20 \rangle$

$$\mathbf{v} = \langle -20 - (-100), 70 - 0 \rangle = \langle 80, 70 \rangle$$

$$\mathbf{u} - \mathbf{v} = \langle 70 - 80, 20 - 70 \rangle = \langle -10, -50 \rangle$$

$$\mathbf{v} - \mathbf{u} = \langle 80 - 70, 70 - 20 \rangle = \langle 10, 50 \rangle$$

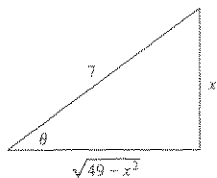
$$\begin{aligned}
 111. \left(\frac{6x^4}{7y^{-2}}\right)(14x^{-1}y^5) &= \frac{12x^4y^5y^2}{x} \\
 &= 12x^3y^7, x \neq 0, y \neq 0
 \end{aligned}$$

$$\begin{aligned}
 113. (18x)^0(4xy)^2(3x^{-1}) &= \frac{16x^2y^2(3)}{x} \\
 &= 48xy^2, x \neq 0
 \end{aligned}$$

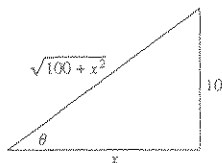
$$115. (2.1 \times 10^9)(3.4 \times 10^{-4}) = 7.14 \times 10^5$$

$$116. (6.5 \times 10^6)(3.8 \times 10^4) = 24.7 \times 10^{10} = 2.47 \times 10^{11}$$

$$117. \sin \theta = \frac{x}{7} \Rightarrow \sqrt{49 - x^2} = 7 \cos \theta$$



$$119. \cot \theta = \frac{x}{10} \Rightarrow \sqrt{x^2 + 100} = 10 \cdot \csc \theta$$



$$121. \cos x(\cos x + 1) = 0$$

$$\cos x = 0 \Rightarrow x = \frac{\pi}{2} + n\pi$$

$$\cos x = -1 \Rightarrow x = \pi + 2n\pi$$

$$123. 3 \sec x + 4 = 10$$

$$\sec x = 2$$

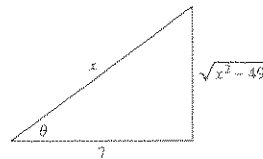
$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$$

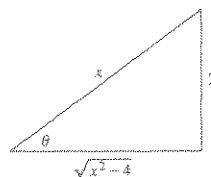
$$112. (5s^5t^{-5})\left(\frac{3s^{-2}}{50t^{-1}}\right) = \frac{3s^3}{10t^4}, s \neq 0$$

$$\begin{aligned}
 114. (5ab^2)(a^{-3}b^0)(2a^0b)^{-2} &= (5ab^2) \frac{1}{a^3} \frac{1}{(2b)^2} \\
 &= \frac{5}{4a^2}, b \neq 0
 \end{aligned}$$

$$118. \sqrt{x^2 - 49} = 7 \tan \theta$$



$$120. \sqrt{x^2 - 4} = 2 \cot \theta$$



$$122. \sin x(2 \sin x + \sqrt{2}) = 0$$

$$\sin x = 0 \text{ or } \sin x = \frac{-\sqrt{2}}{2}$$

$$x = n\pi \text{ or } x = \frac{5\pi}{4} + 2n\pi$$

$$x = \frac{7\pi}{4} + 2n\pi$$

$$124. \cos x \cot x - \cos x = 0$$

$$\cos x(\cot x - 1) = 0$$

$$\cos x = 0 \text{ or } \cot x = 1$$

$$x = \frac{\pi}{2} + n\pi \text{ or } x = \frac{\pi}{4} + n\pi$$

Section 6.4 Vectors and Dot Products

- Know the definition of the dot product of $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$.

$$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2$$

- Know the following properties of the dot product:

1. $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
2. $\mathbf{0} \cdot \mathbf{v} = 0$
3. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$
4. $\mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$
5. $c(\mathbf{u} \cdot \mathbf{v}) = c\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot c\mathbf{v}$

- If θ is the angle between two nonzero vectors \mathbf{u} and \mathbf{v} , then

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$$

- The vectors \mathbf{u} and \mathbf{v} are orthogonal if $\mathbf{u} \cdot \mathbf{v} = 0$.

- Know the definition of vector components. $\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2$ where \mathbf{w}_1 and \mathbf{w}_2 are orthogonal, and \mathbf{w}_1 is parallel to \mathbf{v} . \mathbf{w}_1 is called the projection of \mathbf{u} onto \mathbf{v} and is denoted by

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}} \mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v}.$$

Then we have $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1$.

- Know the definition of work.

1. Projection form: $W = \|\text{proj}_{\overrightarrow{PQ}} \mathbf{F}\| \|\overrightarrow{PQ}\|$
2. Dot product form: $W = \mathbf{F} \cdot \overrightarrow{PQ}$

Vocabulary Check

1. dot product
2. $\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$
3. orthogonal
4. $\left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v}$
5. $\|\text{proj}_{\overrightarrow{PQ}} \mathbf{F}\| \|\overrightarrow{PQ}\|, \mathbf{F} \cdot \overrightarrow{PQ}$

1. $\mathbf{u} \cdot \mathbf{v} = \langle 6, 3 \rangle \cdot \langle 2, -4 \rangle = 6(2) + 3(-4) = 0$

2. $\mathbf{u} \cdot \mathbf{v} = \langle -4, 1 \rangle \cdot \langle 2, -3 \rangle$

$$= (-4)(2) + 1(-3) = -11$$

3. $\mathbf{u} \cdot \mathbf{v} = \langle 5, 1 \rangle \cdot \langle 3, -1 \rangle = 5(3) + 1(-1) = 14$

4. $\mathbf{u} \cdot \mathbf{v} = \langle 3, 2 \rangle \cdot \langle -2, 1 \rangle = 3(-2) + 2(1) = -4$

5. $\mathbf{u} = \langle 2, 2 \rangle$

6. $\mathbf{v} \cdot \mathbf{w} = \langle -3, 4 \rangle \cdot \langle 1, -4 \rangle$

$$\mathbf{u} \cdot \mathbf{u} = 2(2) + 2(2) = 8, \text{ scalar}$$

$$= (-3)(1) + 4(-4) = -19, \text{ scalar}$$

7. $\mathbf{u} = \langle 2, 2 \rangle, \mathbf{v} = \langle -3, 4 \rangle$

8. $4\mathbf{u} \cdot \mathbf{v} = 4\langle 2, 2 \rangle \cdot \langle -3, 4 \rangle$

$$\mathbf{u} \cdot 2\mathbf{v} = 2\mathbf{u} \cdot \mathbf{v} = 4(-3) + 4(4) = 4, \text{ scalar}$$

$$= 4(-6 + 8)$$

$$= 8, \text{ scalar}$$

$$\begin{aligned}
 9. (3\mathbf{w} \cdot \mathbf{v})\mathbf{u} &= (3\langle 1, -4 \rangle \cdot \langle -3, 4 \rangle)\langle 2, 2 \rangle \\
 &= (3(-3) + (-12)(4))\langle 2, 2 \rangle \\
 &= -57\langle 2, 2 \rangle \\
 &= \langle -114, -114 \rangle, \text{ vector}
 \end{aligned}$$

$$\begin{aligned}
 10. (\mathbf{u} \cdot 2\mathbf{v})\mathbf{w} &= (\langle 2, 2 \rangle \cdot 2\langle -3, 4 \rangle)\langle 1, -4 \rangle \\
 &= (2(-6) + 2(8))\langle 1, -4 \rangle \\
 &= 4\langle 1, -4 \rangle \\
 &= \langle 4, -16 \rangle, \text{ vector}
 \end{aligned}$$

$$\begin{aligned}
 11. \mathbf{u} &= \langle -5, 12 \rangle \\
 \|\mathbf{u}\| &= \sqrt{\mathbf{u} \cdot \mathbf{u}} = \sqrt{(-5)^2 + 12^2} = 13
 \end{aligned}$$

$$\begin{aligned}
 12. \mathbf{u} &= \langle 2, -4 \rangle \\
 \|\mathbf{u}\| &= \sqrt{\mathbf{u} \cdot \mathbf{u}} \\
 &= \sqrt{2(2) + (-4)(-4)} \\
 &= \sqrt{20} = 2\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 13. \mathbf{u} &= 20\mathbf{i} + 25\mathbf{j} \\
 \|\mathbf{u}\| &= \sqrt{\mathbf{u} \cdot \mathbf{u}} = \sqrt{(20)^2 + (25)^2} = \sqrt{1025} = 5\sqrt{41}
 \end{aligned}$$

$$\begin{aligned}
 14. \mathbf{u} &= 6\mathbf{i} - 10\mathbf{j} \\
 \|\mathbf{u}\| &= \sqrt{\mathbf{u} \cdot \mathbf{u}} = \sqrt{6(6) + (-10)(-10)} = \sqrt{136} = 2\sqrt{34}
 \end{aligned}$$

$$\begin{aligned}
 15. \mathbf{u} &= -4\mathbf{j} \\
 \|\mathbf{u}\| &= \sqrt{\mathbf{u} \cdot \mathbf{u}} = \sqrt{(-4)(-4)} = 4
 \end{aligned}$$

$$\begin{aligned}
 16. \mathbf{u} &= 9\mathbf{i} \\
 \|\mathbf{u}\| &= \sqrt{\mathbf{u} \cdot \mathbf{u}} = \sqrt{9(9) + 0} = \sqrt{81} = 9
 \end{aligned}$$

$$\begin{aligned}
 17. \mathbf{u} &= \langle -1, 0 \rangle, \mathbf{v} = \langle 0, 2 \rangle \\
 \cos \theta &= \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{0}{(1)(2)} = 0 \Rightarrow \theta = 90^\circ
 \end{aligned}$$

$$18. \mathbf{u} = \langle 4, 4 \rangle, \mathbf{v} = \langle -2, 0 \rangle$$

$$\begin{aligned}
 \cos \theta &= \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} \\
 &= \frac{4(-2) + 4(0)}{(4\sqrt{2})(2)} \\
 &= -\frac{\sqrt{2}}{2} \\
 \theta &= 135^\circ
 \end{aligned}$$

$$\begin{aligned}
 19. \mathbf{u} &= 3\mathbf{i} + 4\mathbf{j}, \mathbf{v} = -2\mathbf{i} + 3\mathbf{j} \\
 \cos \theta &= \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{-6 + 12}{(5)(\sqrt{13})} = \frac{6}{5\sqrt{13}} \\
 \theta &= \arccos\left(\frac{6}{5\sqrt{13}}\right) \approx 70.56^\circ
 \end{aligned}$$

$$\begin{aligned}
 20. \mathbf{u} &= 2\mathbf{i} - 3\mathbf{j}, \mathbf{v} = \mathbf{i} - 2\mathbf{j} \\
 \cos \theta &= \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} \\
 &= \frac{2(1) + (-3)(-2)}{\sqrt{2^2 + 3^2}\sqrt{1^2 + 2^2}} \\
 &= \frac{8}{\sqrt{65}} \approx 0.992278 \\
 \theta &\approx 7.13^\circ
 \end{aligned}$$

$$\begin{aligned}
 21. \mathbf{u} &= 2\mathbf{i}, \mathbf{v} = -3\mathbf{j} \\
 \cos \theta &= \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{0}{(2)(3)} = 0 \Rightarrow \theta = 90^\circ
 \end{aligned}$$

$$22. \mathbf{u} \cdot \mathbf{v} = \langle 0, 4 \rangle \cdot \langle -3, 0 \rangle = 0 \Rightarrow \theta = 90^\circ$$

$$23. \quad \mathbf{u} = \left(\cos \frac{\pi}{3}\right)\mathbf{i} + \left(\sin \frac{\pi}{3}\right)\mathbf{j} = \frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$$

$$\mathbf{v} = \left(\cos \frac{3\pi}{4}\right)\mathbf{i} + \left(\sin \frac{3\pi}{4}\right)\mathbf{j} = -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$$

$$\|\mathbf{u}\| = \|\mathbf{v}\| = 1$$

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \mathbf{u} \cdot \mathbf{v} = \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{2} + \sqrt{6}}{4}$$

$$\theta = \arccos\left(\frac{-\sqrt{2} + \sqrt{6}}{4}\right) = 75^\circ = \frac{5\pi}{12}$$

$$24. \quad \mathbf{u} = \cos\left(\frac{\pi}{4}\right)\mathbf{i} + \sin\left(\frac{\pi}{4}\right)\mathbf{j} = \frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$$

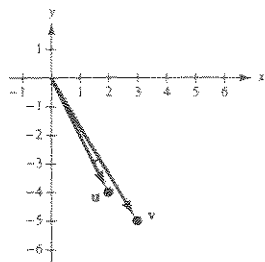
$$\mathbf{v} = \cos\left(\frac{2\pi}{3}\right)\mathbf{i} + \sin\left(\frac{2\pi}{3}\right)\mathbf{j} = -\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$$

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{-(\sqrt{2}/4) + (\sqrt{6}/4)}{(1)(1)} = \frac{\sqrt{6} - \sqrt{2}}{4} \Rightarrow \theta = 75^\circ = \frac{5\pi}{12}$$

$$25. \quad \mathbf{u} = 2\mathbf{i} - 4\mathbf{j}, \mathbf{v} = 3\mathbf{i} - 5\mathbf{j}$$

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{6 + 20}{\sqrt{20}\sqrt{34}}$$

$$= \frac{13\sqrt{170}}{170} \Rightarrow \theta \approx 4.40^\circ$$

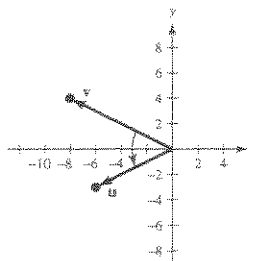


$$26. \quad \mathbf{u} = -6\mathbf{i} - 3\mathbf{j}, \mathbf{v} = -8\mathbf{i} + 4\mathbf{j}$$

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{-6(-8) + (-3)(4)}{\sqrt{45}\sqrt{80}}$$

$$= \frac{36}{60} = 0.6$$

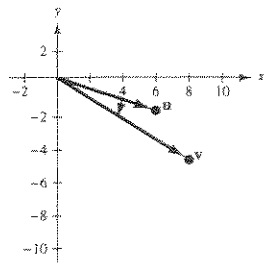
$$\theta \approx 53.13^\circ$$



$$27. \quad \mathbf{u} = 6\mathbf{i} - 2\mathbf{j}, \mathbf{v} = 8\mathbf{i} - 5\mathbf{j}$$

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{48 + 10}{\sqrt{40}\sqrt{89}}$$

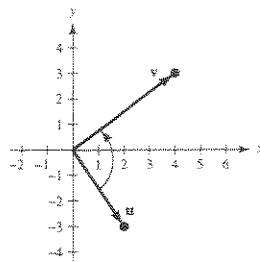
$$= \frac{29\sqrt{890}}{890} \Rightarrow \theta \approx 13.57^\circ$$



$$28. \quad \mathbf{u} = 2\mathbf{i} - 3\mathbf{j}, \mathbf{v} = 4\mathbf{i} + 3\mathbf{j}$$

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{2(4) + (-3)(3)}{\sqrt{13}\sqrt{25}} \approx -0.0555$$

$$\theta \approx 93.18^\circ$$



$$29. P = (1, 2), Q = (3, 4), R = (2, 5)$$

$$\overrightarrow{PQ} = \langle 2, 2 \rangle, \overrightarrow{PR} = \langle 1, 3 \rangle, \overrightarrow{QR} = \langle -1, 1 \rangle$$

$$\cos \alpha = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\|\overrightarrow{PQ}\| \|\overrightarrow{PR}\|} = \frac{8}{(2\sqrt{2})(\sqrt{10})} \Rightarrow \alpha = \arccos \frac{2}{\sqrt{5}} \approx 26.6^\circ$$

$$\cos \beta = \frac{\overrightarrow{PQ} \cdot \overrightarrow{QR}}{\|\overrightarrow{PQ}\| \|\overrightarrow{QR}\|} = 0 \Rightarrow \beta = 90^\circ$$

$$\text{Thus, } \gamma \approx 180^\circ - 26.6^\circ - 90^\circ = 63.4^\circ.$$

$$30. P = (-3, 0), Q = (2, 2), R = (0, 6)$$

$$\overrightarrow{PQ} = \langle 5, 2 \rangle, \overrightarrow{QR} = \langle -2, 4 \rangle, \overrightarrow{PR} = \langle 3, 6 \rangle,$$

$$\overrightarrow{QP} = \langle -5, -2 \rangle$$

$$\cos \alpha = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\|\overrightarrow{PQ}\| \|\overrightarrow{PR}\|} = \frac{27}{(\sqrt{29})(\sqrt{45})} \Rightarrow \alpha \approx 41.6^\circ$$

$$\cos \beta = \frac{\overrightarrow{QR} \cdot \overrightarrow{QP}}{\|\overrightarrow{QR}\| \|\overrightarrow{QP}\|} = \frac{2}{(\sqrt{20})(\sqrt{29})} \Rightarrow \beta \approx 85.2^\circ$$

$$\phi = 180^\circ - 41.6^\circ - 85.2^\circ \approx 53.2^\circ$$

$$31. \mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta$$

$$= (9)(36) \cos \frac{3\pi}{4}$$

$$= 324 \left(-\frac{\sqrt{2}}{2} \right)$$

$$= -162\sqrt{2}$$

$$32. \mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta$$

$$= (4)(12) \cos \frac{\pi}{3} = 48 \left(\frac{1}{2} \right) = 24$$

$$33. \mathbf{u} = \langle -12, 30 \rangle, \mathbf{v} = \left\langle \frac{1}{2}, -\frac{5}{4} \right\rangle$$

$$\mathbf{u} = -24\mathbf{v} \Rightarrow \mathbf{u} \text{ and } \mathbf{v} \text{ are parallel.}$$

$$34. \mathbf{u} = \langle 15, 45 \rangle, \mathbf{v} = \langle -5, 12 \rangle$$

$$\mathbf{u} \neq k\mathbf{v} \Rightarrow \text{Not parallel}$$

$$\mathbf{u} \cdot \mathbf{v} \neq 0 \Rightarrow \text{Not orthogonal}$$

Neither

$$35. \mathbf{u} = \frac{1}{4}(3\mathbf{i} - \mathbf{j}), \mathbf{v} = 5\mathbf{i} + 6\mathbf{j}$$

$$\mathbf{u} \neq k\mathbf{v} \Rightarrow \text{Not parallel}$$

$$\mathbf{u} \cdot \mathbf{v} \neq 0 \Rightarrow \text{Not orthogonal}$$

Neither

$$36. \mathbf{u} = \mathbf{j}, \mathbf{v} = \mathbf{i} - 2\mathbf{j}$$

$$\mathbf{u} \neq k\mathbf{v} \Rightarrow \text{Not parallel}$$

$$\mathbf{u} \cdot \mathbf{v} \neq 0 \Rightarrow \text{Not orthogonal}$$

Neither

$$37. \mathbf{u} = 2\mathbf{i} - 2\mathbf{j}, \mathbf{v} = -\mathbf{i} - \mathbf{j}$$

$$\mathbf{u} \cdot \mathbf{v} = 0 \Rightarrow \mathbf{u} \text{ and } \mathbf{v} \text{ are orthogonal.}$$

$$38. -4\mathbf{v} = -4(-2\mathbf{i} - \mathbf{j}) = 8\mathbf{i} + 4\mathbf{j} = \mathbf{u} \Rightarrow \text{parallel}$$

$$39. \mathbf{u} \cdot \mathbf{v} = \langle 2, -k \rangle \cdot \langle 3, 2 \rangle = 6 - 2k = 0 \Rightarrow k = 3$$

$$40. \mathbf{u} \cdot \mathbf{v} = \langle 3, 2 \rangle \cdot \langle 2, -k \rangle = 6 - 2k = 0 \Rightarrow k = 3$$

$$41. \mathbf{u} \cdot \mathbf{v} = \langle 1, 4 \rangle \cdot \langle 2k, -5 \rangle = 2k - 20 = 0 \Rightarrow k = 10$$

$$42. \mathbf{u} \cdot \mathbf{v} = \langle -3k, 5 \rangle \cdot \langle 2, -4 \rangle = -6k - 20 = 0 \Rightarrow k = -\frac{10}{3}$$

$$43. \mathbf{u} \cdot \mathbf{v} = \langle -3k, 2 \rangle \cdot \langle -6, 0 \rangle = 18k = 0 \Rightarrow k = 0$$

$$44. \mathbf{u} \cdot \mathbf{v} = \langle 4, -4k \rangle \cdot \langle 0, 3 \rangle = -12k = 0 \Rightarrow k = 0$$

45. $\mathbf{u} = \langle 3, 4 \rangle$, $\mathbf{v} = \langle 8, 2 \rangle$

$$\begin{aligned}\mathbf{w}_1 &= \text{proj}_{\mathbf{v}} \mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v} \\ &= \left(\frac{32}{68} \right) \mathbf{v} = \frac{8}{17} \langle 8, 2 \rangle = \frac{16}{17} \langle 4, 1 \rangle\end{aligned}$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 3, 4 \rangle - \frac{16}{17} \langle 4, 1 \rangle = \frac{13}{17} \langle -1, 4 \rangle$$

$$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = \frac{16}{17} \langle 4, 1 \rangle + \frac{13}{17} \langle -1, 4 \rangle$$

47. $\mathbf{u} = \langle 0, 3 \rangle$, $\mathbf{v} = \langle 2, 15 \rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}} \mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v} = \frac{45}{229} \langle 2, 15 \rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 0, 3 \rangle - \frac{45}{229} \langle 2, 15 \rangle$$

$$= \left\langle -\frac{90}{229}, \frac{12}{229} \right\rangle = \frac{6}{229} \langle -15, 2 \rangle$$

$$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = \frac{45}{229} \langle 2, 15 \rangle + \frac{6}{229} \langle -15, 2 \rangle$$

49. $\text{proj}_{\mathbf{v}} \mathbf{u} = \mathbf{u}$ since they are parallel.

$$\begin{aligned}\text{proj}_{\mathbf{v}} \mathbf{u} &= \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \mathbf{v} \\ &= \frac{18 + 8}{36 + 16} \mathbf{v} = \frac{26}{52} \langle 6, 4 \rangle = \langle 3, 2 \rangle = \mathbf{u}\end{aligned}$$

51. $\text{proj}_{\mathbf{v}} \mathbf{u} = \mathbf{0}$ since they are perpendicular.

$$\text{proj}_{\mathbf{v}} \mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \mathbf{v} = \mathbf{0}, \text{ since } \mathbf{u} \cdot \mathbf{v} = 0.$$

53. $\mathbf{u} = \langle 2, 6 \rangle$

For \mathbf{v} to be orthogonal to \mathbf{u} , $\mathbf{u} \cdot \mathbf{v}$ must equal 0.

Two possibilities: $\langle 6, -2 \rangle$ and $\langle -6, 2 \rangle$

55. $\mathbf{u} = \frac{1}{2}\mathbf{i} - \frac{3}{4}\mathbf{j}$

For \mathbf{v} to be orthogonal to \mathbf{u} , $\mathbf{u} \cdot \mathbf{v}$ must equal 0.

Two possibilities: $\langle \frac{3}{4}, \frac{1}{2} \rangle$ and $\langle -\frac{3}{4}, -\frac{1}{2} \rangle$

46. $\mathbf{u} = \langle 4, 2 \rangle$, $\mathbf{v} = \langle 1, -2 \rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}} \mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v} = 0 \langle 1, -2 \rangle = \langle 0, 0 \rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 4, 2 \rangle - \langle 0, 0 \rangle = \langle 4, 2 \rangle$$

$$\mathbf{u} = \langle 4, 2 \rangle + \langle 0, 0 \rangle$$

48. $\mathbf{u} = \langle -5, -1 \rangle$, $\mathbf{v} = \langle -1, 1 \rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}} \mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v} = \frac{4}{2} \langle -1, 1 \rangle = 2 \langle -1, 1 \rangle$$

$$\begin{aligned}\mathbf{w}_2 &= \mathbf{u} - \mathbf{w}_1 = \langle -5, -1 \rangle - 2 \langle -1, 1 \rangle \\ &= \langle -3, -3 \rangle = 3 \langle -1, -1 \rangle\end{aligned}$$

$$\mathbf{u} = \langle -3, -3 \rangle + \langle -2, 2 \rangle$$

50. Because \mathbf{u} and \mathbf{v} are parallel, the projection of \mathbf{u} onto \mathbf{v} is \mathbf{u} .

52. Because \mathbf{u} and \mathbf{v} are orthogonal, the projection of \mathbf{u} onto \mathbf{v} is $\mathbf{0}$.

54. For \mathbf{v} to be orthogonal to $\mathbf{u} = \langle -7, 5 \rangle$, their dot product must be zero.

Two possibilities: $\langle 5, 7 \rangle$, $\langle -5, -7 \rangle$

56. $\mathbf{u} = -\frac{5}{2}\mathbf{i} - 3\mathbf{j}$

For \mathbf{v} to be orthogonal to \mathbf{u} , $\mathbf{u} \cdot \mathbf{v}$ must be equal to 0.

Two possibilities: $\mathbf{v} = 3\mathbf{i} - \frac{5}{2}\mathbf{j}$ and $\mathbf{v} = -3\mathbf{i} + \frac{5}{2}\mathbf{j}$

57. $W = \|\text{proj}_{\overrightarrow{PQ}} \mathbf{v}\| \|\overrightarrow{PQ}\|$ where $\overrightarrow{PQ} = \langle 4, 7 \rangle$ and $\mathbf{v} = \langle 1, 4 \rangle$

$$\text{proj}_{\overrightarrow{PQ}} \mathbf{v} = \left(\frac{\mathbf{v} \cdot \overrightarrow{PQ}}{\|\overrightarrow{PQ}\|^2} \right) \overrightarrow{PQ} = \left(\frac{32}{65} \right) \langle 4, 7 \rangle$$

$$W = \|\text{proj}_{\overrightarrow{PQ}} \mathbf{v}\| \|\overrightarrow{PQ}\| = \left(\frac{32\sqrt{65}}{65} \right) (\sqrt{65}) = 32$$

59. (a) $\mathbf{u} \cdot \mathbf{v} = \langle 1245, 2600 \rangle \cdot \langle 12.20, 8.50 \rangle$
 $= 1245(12.20) + 2600(8.50) = 37,289$

This is the total dollar value of the picture frames produced.

- (b) Multiply \mathbf{v} by 1.02.

58. $P = (1, 3)$, $Q = (-3, 5)$, $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$

$$\text{work} = \|\text{proj}_{\overrightarrow{PQ}} \mathbf{v}\| \|\overrightarrow{PQ}\| \text{ where}$$

$$\overrightarrow{PQ} = \langle -4, 2 \rangle \text{ and } \mathbf{v} = \langle -2, 3 \rangle.$$

$$\text{proj}_{\overrightarrow{PQ}} \mathbf{v} = \left(\frac{\mathbf{v} \cdot \overrightarrow{PQ}}{\|\overrightarrow{PQ}\|^2} \right) \overrightarrow{PQ} = \left(\frac{14}{20} \right) \langle -4, 2 \rangle$$

$$\text{work} = \|\text{proj}_{\overrightarrow{PQ}} \mathbf{v}\| \|\overrightarrow{PQ}\| = \left(\frac{14\sqrt{20}}{20} \right) (\sqrt{20}) = 14$$

60. $\mathbf{u} = \langle 3240, 2450 \rangle$, $\mathbf{v} = \langle 1.75, 1.25 \rangle$

$$(a) \mathbf{u} \cdot \mathbf{v} = 3240(1.75) + 2450(1.25) = 8732.50$$

This gives the total revenue earned.

- (b) To increase prices by $2\frac{1}{2}$ percent, multiply \mathbf{v} by 1.025:

$$1.025\langle 1.75, 1.25 \rangle, \text{ scalar multiplication}$$

61. (a) $\mathbf{F} = -30,000\mathbf{j}$, Gravitational force

$$\mathbf{v} = \langle \cos(d^\circ), \sin(d^\circ) \rangle$$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}} \mathbf{F} = \left(\frac{\mathbf{F} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v} = (\mathbf{F} \cdot \mathbf{v}) \mathbf{v} = -30,000 \sin(d^\circ) \langle \cos d^\circ, \sin d^\circ \rangle$$

$$= \langle -30,000 \sin d^\circ \cos d^\circ, -30,000 \sin^2 d^\circ \rangle$$

$$\text{Force needed: } 30,000 \sin(d^\circ)$$

(b)

d	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
Force	0	523.6	1047.0	1570.1	2092.7	2614.7	3135.9	3656.1	4175.2	4693.0	5209.4

$$(c) \mathbf{w}_2 = \mathbf{F} - \mathbf{w}_1 = -30,000\mathbf{j} + 2614.7(\cos(5^\circ)\mathbf{i} + \sin(5^\circ)\mathbf{j}) \approx 2604.75\mathbf{i} - 29,772.11\mathbf{j}$$

$$\|\mathbf{w}_2\| \approx 29,885.8 \text{ pounds}$$

62. $\mathbf{F} = -5400\mathbf{j}$, Gravitational force

$$\mathbf{v} = \cos 10^\circ \mathbf{i} + \sin 10^\circ \mathbf{j}$$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}} \mathbf{F} = \left(\frac{\mathbf{F} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v} = (\mathbf{F} \cdot \mathbf{v}) \mathbf{v} \approx -937.7\mathbf{v}$$

$$\text{Magnitude: } 937.7 \text{ pounds}$$

$$\mathbf{w}_2 = \mathbf{F} - \mathbf{w}_1$$

$$= -5400\mathbf{j} + 937.7[\cos 10^\circ \mathbf{i} + \sin 10^\circ \mathbf{j}]$$

$$\approx 923.45\mathbf{i} - 5237.17\mathbf{j}$$

$$\|\mathbf{w}_2\| \approx 5318 \text{ pounds}$$

63. (a) $\mathbf{F} = 15,691\langle \cos 30^\circ, \sin 30^\circ \rangle$

$$\overrightarrow{PQ} = d\langle 1, 0 \rangle$$

$$W = \mathbf{F} \cdot \overrightarrow{PQ} = 15,691 \frac{\sqrt{3}}{2} d \approx 13,588.8d$$

- (b)

d	0	200	400	800
Work	0	2,717,761	5,435,522	10,871,044

64. $W = (45)(20) \cos 30^\circ \approx 779.4 \text{ foot-pounds}$

65. $\|\mathbf{F}\| = 250, \|\vec{PQ}\| = 100, \theta = 30^\circ$

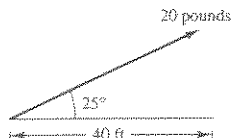
$$\begin{aligned} W &= \|\mathbf{F}\| \|\vec{PQ}\| \cos \theta \\ &= (250)(100) \cos 30^\circ \\ &= 25,000 \frac{\sqrt{3}}{2} \\ &= 12,500\sqrt{3} \text{ foot-pounds} \\ &\approx 21,650.64 \text{ foot-pounds} \end{aligned}$$

66. $\|\mathbf{F}\| = 25, \|\vec{PQ}\| = 50, \theta = 20^\circ$

$$\begin{aligned} W &= \|\mathbf{F}\| \|\vec{PQ}\| \cos \theta \\ &= (25)(50) \cos 20^\circ \\ &\approx 1174.62 \text{ foot-pounds} \end{aligned}$$

67. $W = (\cos 25^\circ)(20)(40) \approx 725.05 \text{ foot-pounds}$

68. $\text{Work} = (\cos 20^\circ)(25)(12) \approx 281.9 \text{ foot-pounds}$



69. True. $\mathbf{u} \cdot \mathbf{v} = 0$

70. False. Work is a scalar, not a vector.

71. $\mathbf{u} \cdot \mathbf{v} = 0 \Rightarrow$ they are orthogonal (unit vectors).

72. (a) $\mathbf{u} \cdot \mathbf{v} = 0 \Rightarrow \mathbf{u}$ and \mathbf{v} are orthogonal and $\theta = \frac{\pi}{2}$.

73. (a) $\text{proj}_{\mathbf{v}} \mathbf{u} = \mathbf{u} \Rightarrow \mathbf{u}$ and \mathbf{v} are parallel.

(b) $\mathbf{u} \cdot \mathbf{v} > 0 \Rightarrow \cos \theta > 0 \Rightarrow 0 \leq \theta < \frac{\pi}{2}$

(b) $\text{proj}_{\mathbf{v}} \mathbf{u} = \mathbf{0} \Rightarrow \mathbf{u}$ and \mathbf{v} are orthogonal.

(c) $\mathbf{u} \cdot \mathbf{v} < 0 \Rightarrow \cos \theta < 0 \Rightarrow \frac{\pi}{2} < \theta \leq \pi$

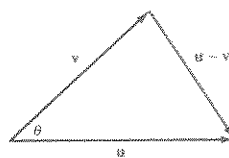
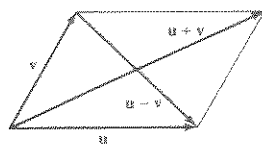
74. Let \mathbf{u} and \mathbf{v} be two adjacent sides of the rhombus, $\|\mathbf{u}\| = \|\mathbf{v}\|$. The diagonals are $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.

75. Use the Law of Cosines on the triangle:

$$\begin{aligned} (\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) &= \mathbf{u} \cdot \mathbf{u} + \mathbf{v} \cdot \mathbf{u} - \mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{v} \\ &= \|\mathbf{u}\|^2 - \|\mathbf{v}\|^2 = 0 \end{aligned}$$

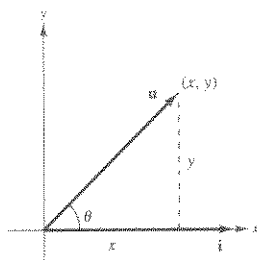
$$\begin{aligned} \|\mathbf{u} - \mathbf{v}\|^2 &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\|\mathbf{u}\|\|\mathbf{v}\|\cos \theta \\ &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\mathbf{u} \cdot \mathbf{v} \end{aligned}$$

Hence, the diagonals are perpendicular.



76. $\mathbf{u} \cdot (c\mathbf{v} + d\mathbf{w}) = c(\mathbf{u} \cdot \mathbf{v}) + d(\mathbf{u} \cdot \mathbf{w}) = c0 + d0 = 0$

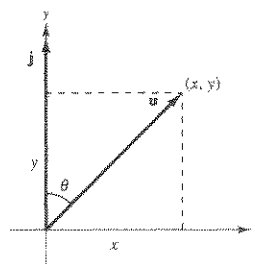
77. From trigonometry, $x = \cos \theta$ and $y = \sin \theta$. Thus, $\mathbf{u} = \langle x, y \rangle = \cos \theta \mathbf{i} + \sin \theta \mathbf{j}$.



78. From trigonometry,

$$x = \cos\left(\frac{\pi}{2} - \theta\right) \text{ and } y = \sin\left(\frac{\pi}{2} - \theta\right).$$

$$\text{Thus, } \mathbf{u} = \langle x, y \rangle = \cos\left(\frac{\pi}{2} - \theta\right)\mathbf{i} + \sin\left(\frac{\pi}{2} - \theta\right)\mathbf{j}.$$



79. $g(x) = f(x - 4)$ is a horizontal shift of f four units to the right.

80. g is a reflection of f with respect to the x -axis.

81. $g(x) = f(x) + 6$ is a vertical shift of f six units upward.

82. g is a horizontal shrink of f .

83. $\sqrt{-4} - 1 = 2i - 1 = -1 + 2i$

84. $\sqrt{-8} + 5 = 2\sqrt{2}i + 5 = 5 + 2\sqrt{2}i$

85. $3i(4 - 5i) = 12i + 15 = 15 + 12i$

86. $-2i(1 + 6i) = -2i + 12 = 12 - 2i$

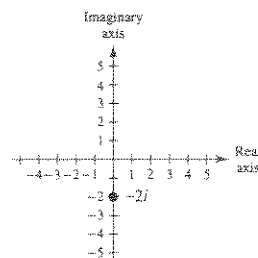
87. $(1 + 3i)(1 - 3i) = 1 - (3i)^2 = 1 + 9 = 10$

88. $(7 - 4i)(7 + 4i) = 49 + 16 = 65$

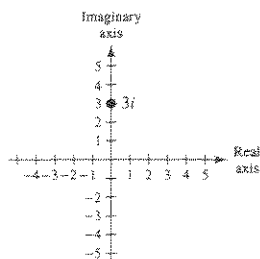
$$\begin{aligned} 89. \frac{3}{1+i} + \frac{2}{2-3i} &= \frac{3}{1+i} \cdot \frac{1-i}{1-i} + \frac{2}{2-3i} \cdot \frac{2+3i}{2+3i} \\ &= \frac{3-3i}{2} + \frac{4+6i}{13} \\ &= \frac{39-39i+8+12i}{26} \\ &= \frac{47}{26} - \frac{27}{26}i \end{aligned}$$

$$\begin{aligned} 90. \frac{6}{4-i} \frac{4+i}{4+i} - \frac{3}{1+i} \cdot \frac{1-i}{1-i} &= \frac{24+6i}{17} - \frac{3-3i}{2} \\ &= \frac{48+12i-51+51i}{34} \\ &= \frac{-3}{34} + \frac{63}{34}i \end{aligned}$$

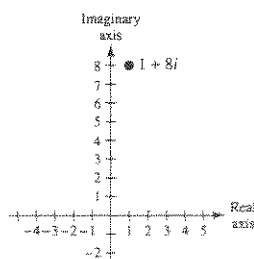
91. $-2i$



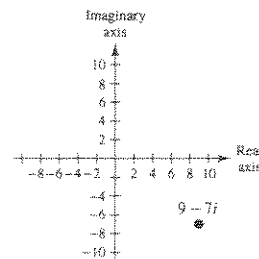
92. $3i$



93. $1 + 8i$



94. $9 - 7i$



Section 6.5 Trigonometric Form of a Complex Number

- You should be able to graphically represent complex numbers.
- The absolute value of the complex numbers $z = a + bi$ is $|z| = \sqrt{a^2 + b^2}$.
- The trigonometric form of the complex number $z = a + bi$ is $z = r(\cos \theta + i \sin \theta)$ where
 - (a) $a = r \cos \theta$ (b) $b = r \sin \theta$
 - (c) $r = \sqrt{a^2 + b^2}$; r is called the modulus of z . (d) $\tan \theta = b/a$; θ is called the argument of z .
- Given $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$:
 - (a) $z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$
 - (b) $\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)]$, $z_2 \neq 0$
- You should know DeMoivre's Theorem: If $z = r(\cos \theta + i \sin \theta)$, then for any positive integer n ,

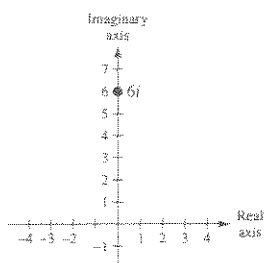
$$z^n = r^n(\cos n\theta + i \sin n\theta).$$
- You should know that for any positive integer n , $z = r(\cos \theta + i \sin \theta)$ has n distinct n th roots given by

$$\sqrt[n]{r} \left[\cos \left(\frac{\theta + 2\pi k}{n} \right) + i \sin \left(\frac{\theta + 2\pi k}{n} \right) \right]$$
 where $k = 0, 1, 2, \dots, n-1$.

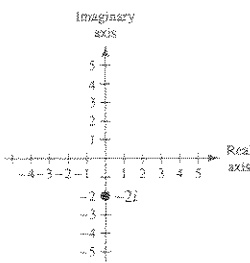
Vocabulary Check

1. absolute value
2. trigonometric form, modulus, argument
3. DeMoivre's
4. n th root

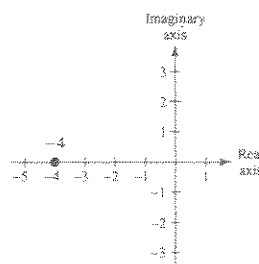
1. $|6i| = 6$



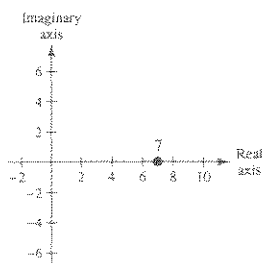
2. $|-2i| = 2$



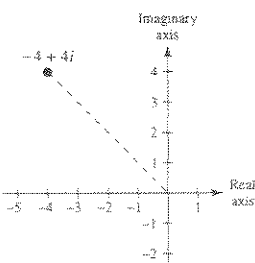
3. $|-4| = \sqrt{(-4)^2 + 0^2} = \sqrt{16} = 4$



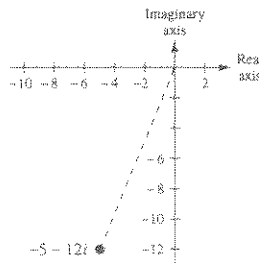
4. $|7| = 7$



5. $|-4 + 4i| = \sqrt{(-4)^2 + (4)^2} = \sqrt{32} = 4\sqrt{2}$

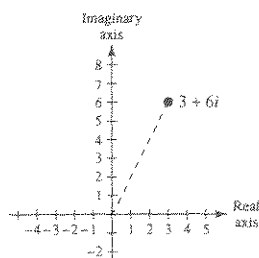


6. $|-5 - 12i| = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$



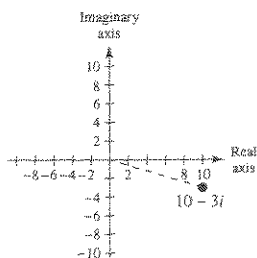
$$7. |3 + 6i| = \sqrt{9 + 36}$$

$$= \sqrt{45} = 3\sqrt{5}$$



$$8. |10 - 3i| = \sqrt{10^2 + (-3)^2}$$

$$= \sqrt{109}$$



$$9. z = 3i$$

$$r = \sqrt{0^2 + 3^2} = \sqrt{9} = 3$$

$$\tan \theta = \frac{3}{0}, \text{undefined} \Rightarrow \theta = \frac{\pi}{2}$$

$$z = 3\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$$

$$10. z = 4$$

$$r = \sqrt{4^2 + 0^2} = \sqrt{16} = 4$$

$$\tan \theta = \frac{0}{4} = 0 \Rightarrow \theta = 0$$

$$z = 4(\cos 0 + i \sin 0)$$

$$11. z = -2$$

$$r = \sqrt{(-2)^2 + 0^2} = 2$$

$$\tan \theta = \pi \Rightarrow \theta = \pi$$

$$z = 2(\cos \pi + i \sin \pi)$$

$$12. z = -i$$

$$r = \sqrt{0^2 + (-1)^2} = 1$$

$$\tan \theta \text{ undefined} \Rightarrow \theta = \frac{3\pi}{2}$$

$$z = \cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}$$

$$13. z = -2 - 2i$$

$$r = \sqrt{(-2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$$

$$\tan \theta = \frac{-2}{-2} = 1, \theta \text{ is in Quadrant III.}$$

$$\theta = \frac{5\pi}{4}$$

$$z = 2\sqrt{2}\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right)$$

$$14. z = 3 + 3i$$

$$r = \sqrt{3^2 + 3^2} = 3\sqrt{2}$$

$$\tan \theta = \frac{3}{3} = 1 \Rightarrow \theta = \frac{\pi}{4}$$

$$z = 3\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$$

$$15. z = \sqrt{3} - i$$

$$r = \sqrt{(\sqrt{3})^2 + (-1)^2} = 2$$

$$\tan \theta = \frac{-1}{\sqrt{3}} \Rightarrow \theta = \frac{11\pi}{6}$$

$$z = 2\left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right)$$

$$16. z = -1 + \sqrt{3}i$$

$$r = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{4} = 2$$

$$\tan \theta = \frac{\sqrt{3}}{-1} = -\sqrt{3} \Rightarrow \theta = \frac{2\pi}{3}$$

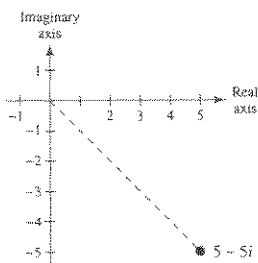
$$z = 2\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$$

$$17. z = 5 - 5i$$

$$r = \sqrt{5^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}$$

$$\tan \theta = \frac{-5}{5} = -1 \Rightarrow \theta = \frac{7\pi}{4}$$

$$z = 5\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$$

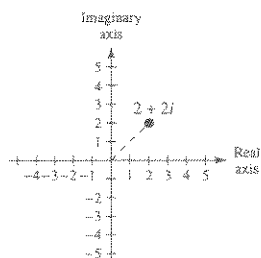


18. $z = 2 + 2i$

$$r = \sqrt{4 + 4} = 2\sqrt{2}$$

$$\tan \theta = \frac{2}{2} = 1 \Rightarrow \theta = \frac{\pi}{4}$$

$$z = 2\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

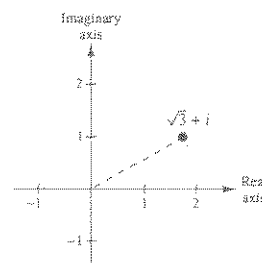


19. $z = \sqrt{3} + i$

$$r = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$$

$$\tan \theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow \theta = \frac{\pi}{6}$$

$$z = 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

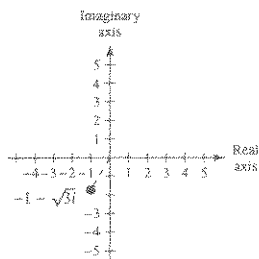


20. $z = -1 - \sqrt{3}i$

$$r = \sqrt{1 + 3} = 2$$

$$\tan \theta = \frac{\sqrt{3}}{1} \Rightarrow \theta = 240^\circ = \frac{4\pi}{3}$$

$$z = 2 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right)$$

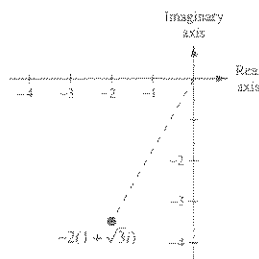


21. $z = -2(1 + \sqrt{3}i)$

$$r = \sqrt{(-2)^2 + (-2\sqrt{3})^2} = \sqrt{16} = 4$$

$$\tan \theta = \frac{\sqrt{3}}{1} = \sqrt{3} \Rightarrow \theta = \frac{4\pi}{3}$$

$$z = 4 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right)$$

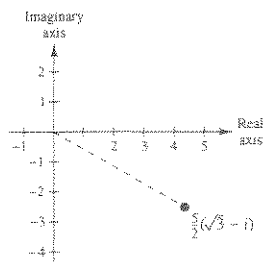


22. $z = \frac{5}{2}(\sqrt{3} - i)$

$$r = \sqrt{\left(\frac{5}{2}\sqrt{3}\right)^2 + \left(\frac{5}{2}(-1)\right)^2} = \sqrt{\frac{100}{4}} = \sqrt{25} = 5$$

$$\tan \theta = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3} \Rightarrow \theta = \frac{11\pi}{6}$$

$$z = 5 \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right)$$

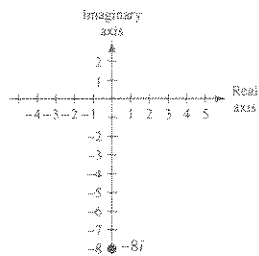


23. $z = -8i$

$$r = \sqrt{0 + (-8)^2} = \sqrt{64} = 8$$

$$\tan \theta = \frac{-8}{0}, \text{ undefined} \Rightarrow \theta = \frac{3\pi}{2}$$

$$z = 8 \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right)$$

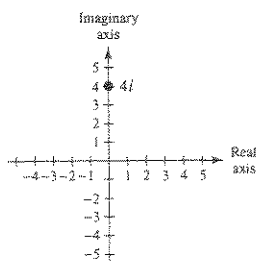


24. $z = 4i$

$r = 4$

$\tan \theta = \frac{4}{0} \text{ undefined} \Rightarrow \theta = \frac{\pi}{2}$

$z = 4\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$

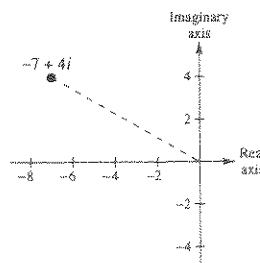


25. $z = -7 + 4i$

$r = \sqrt{49 + 16} = \sqrt{65}$

$\tan \theta = \frac{4}{-7} \Rightarrow \theta \approx 2.62 \text{ radians or } 150.26^\circ$

$z = \sqrt{65}(\cos 150.26^\circ + i \sin 150.26^\circ)$

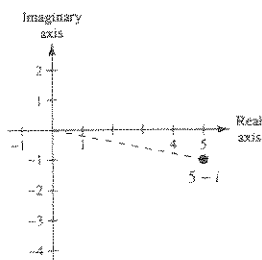


26. $z = 5 - i$

$r = \sqrt{5^2 + (-1)^2} = \sqrt{26}$

$\tan \theta = -\frac{1}{5} \Rightarrow \theta \approx -11.3^\circ \text{ or } 348.7^\circ$

$z \approx \sqrt{26}(\cos(-11.3^\circ) + i \sin(-11.3^\circ))$

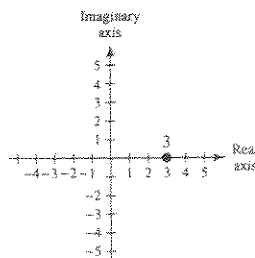


27. $z = 3$

$r = \sqrt{3^2 + 0^2} = 3$

$\tan \theta = \frac{0}{3} = 0 \Rightarrow \theta = 0^\circ$

$z = 3(\cos 0^\circ + i \sin 0^\circ)$

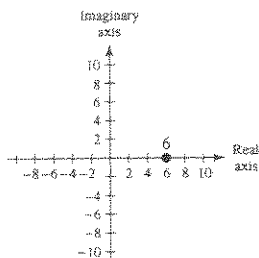


28. $z = 6$

$r = 6$

$\theta = 0$

$z = 6(\cos 0 + i \sin 0)$

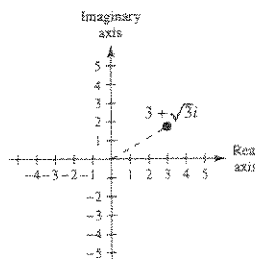


29. $z = 3 + \sqrt{3}i$

$r = \sqrt{9 + 3} = \sqrt{12} = 2\sqrt{3}$

$\tan \theta = \frac{\sqrt{3}}{3} \Rightarrow \theta = \frac{\pi}{6} \text{ or } 30^\circ$

$z = 2\sqrt{3}\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$

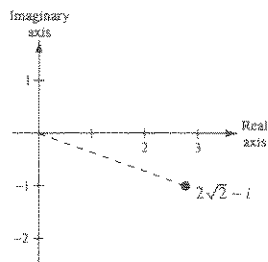


30. $z = 2\sqrt{2} - i$

$$r = \sqrt{(2\sqrt{2})^2 + (-1)^2} = \sqrt{9} = 3$$

$$\tan \theta = \frac{-1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4} \Rightarrow \theta \approx -19.5^\circ \text{ or } 340.5^\circ$$

$$z = 3(\cos(-19.5^\circ) + i \sin(-19.5^\circ))$$

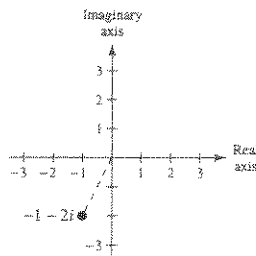


31. $z = -1 - 2i$

$$r = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$\tan \theta = \frac{-2}{-1} = 2 \Rightarrow \theta \approx 243.4^\circ$$

$$z = \sqrt{5}(\cos 243.4^\circ + i \sin 243.4^\circ)$$

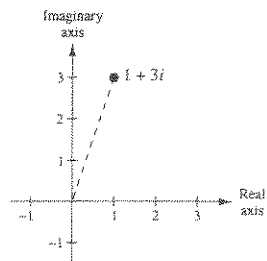


32. $z = 1 + 3i$

$$r = \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$\tan \theta = \frac{3}{1} = 3 \Rightarrow \theta \approx 71.6^\circ$$

$$z \approx \sqrt{10}(\cos 71.6^\circ + i \sin 71.6^\circ)$$

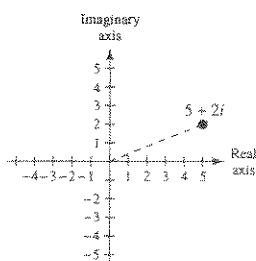


33. $z = 5 + 2i$

$$r = \sqrt{25 + 4} = \sqrt{29} \approx 5.385$$

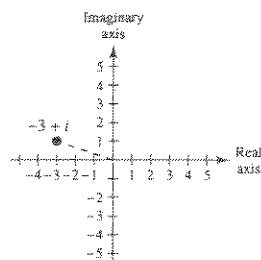
$$\tan \theta = \frac{2}{5} \Rightarrow \theta \approx 21.80^\circ$$

$$z = \sqrt{29}(\cos 21.80^\circ + i \sin 21.80^\circ)$$



34. $-3 + i \approx 3.16(\cos 161.6^\circ + i \sin 161.6^\circ)$

$$= 3.16(\cos 2.82 + i \sin 2.82)$$

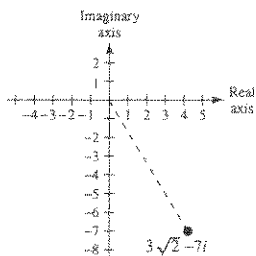


35. $z = 3\sqrt{2} - 7i$

$$r = \sqrt{18 + 49} = \sqrt{67} \approx 8.185$$

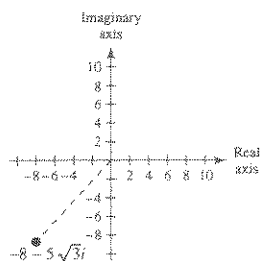
$$\tan \theta = \frac{-7}{3\sqrt{2}} \approx -1.6499 \Rightarrow \theta \approx 301.22^\circ$$

$$z = \sqrt{67}(\cos 301.22^\circ + i \sin 301.22^\circ)$$

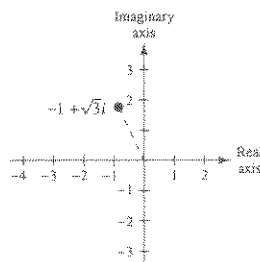


$$36. -8 - 5\sqrt{3}i \approx 11.79(\cos 227.3^\circ + i \sin 227.3^\circ)$$

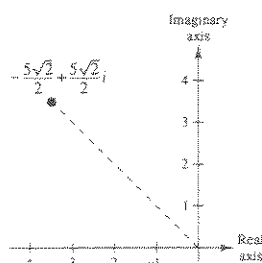
$$-8 - 5\sqrt{3}i = 11.79(\cos 3.97 + i \sin 3.97)$$



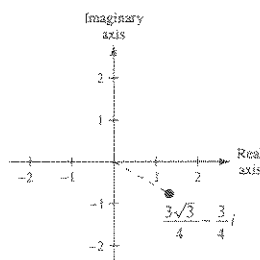
$$37. 2(\cos 120^\circ + i \sin 120^\circ) = 2\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -1 + \sqrt{3}i$$



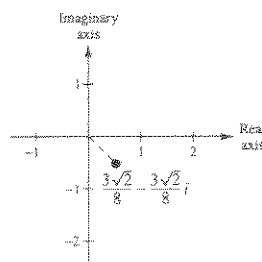
$$38. 5(\cos 135^\circ + i \sin 135^\circ) = 5\left[-\frac{\sqrt{2}}{2} + i\left(\frac{\sqrt{2}}{2}\right)\right] = -\frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}i$$



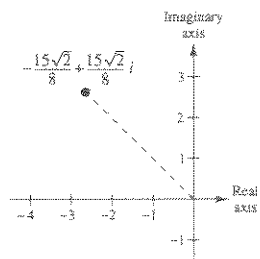
$$39. \frac{3}{2}(\cos 330^\circ + i \sin 330^\circ) = \frac{3}{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \frac{3\sqrt{3}}{4} - \frac{3}{4}i$$



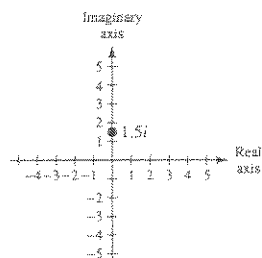
$$40. \frac{3}{4}(\cos 315^\circ + i \sin 315^\circ) = \frac{3}{4}\left[\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right] = \frac{3\sqrt{2}}{8} - \frac{3\sqrt{2}}{8}i$$



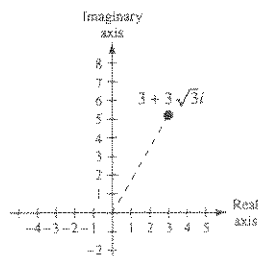
$$41. 3.75\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) = -\frac{15\sqrt{2}}{8} + \frac{15\sqrt{2}}{8}i$$



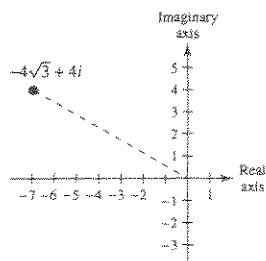
$$42. 1.5\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) = 1.5(0 + i) = 1.5i$$



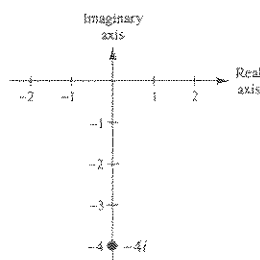
$$43. 6\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right) = 6\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 3 + 3\sqrt{3}i$$



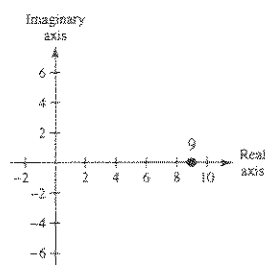
$$44. 8\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right) = 8\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) \\ = -4\sqrt{3} + 4i$$



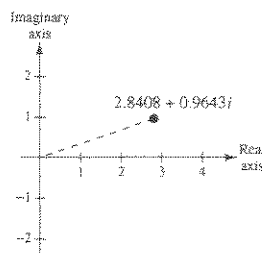
$$45. 4\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right) = 4(0 - i) = -4i$$



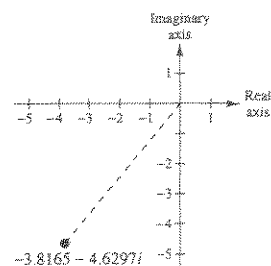
$$46. 9(\cos 0 + i \sin 0) = 9$$



$$47. 3[\cos(18^\circ 45') + i \sin(18^\circ 45')] \approx 2.8408 + 0.9643i$$



$$48. 6[\cos(230^\circ 30') + i \sin(230^\circ 30')] \approx -3.8165 - 4.6297i$$



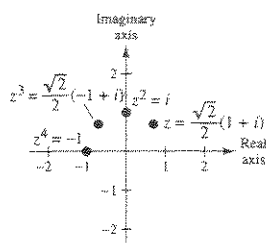
$$49. 5\left(\cos \frac{\pi}{9} + i \sin \frac{\pi}{9}\right) \approx 4.6985 + 1.7101i$$

$$50. 12\left(\cos \frac{3\pi}{5} + i \sin \frac{3\pi}{5}\right) \approx -3.71 + 11.41i$$

$$51. 9(\cos 58^\circ + i \sin 58^\circ) \approx 4.7693 + 7.6324i$$

$$52. 4(\cos 216.5^\circ + i \sin 216.5^\circ) \approx -3.22 - 2.38i$$

53.



The absolute value of each power is 1.

$$54. z = \frac{1}{2}(1 + \sqrt{3}i)$$

$$z^2 = \frac{1}{2}(1 + \sqrt{3}i)\frac{1}{2}(1 + \sqrt{3}i) = \frac{1}{2}(-1 + \sqrt{3}i)$$

$$z^3 = z^2 z = \frac{1}{2}(-1 + \sqrt{3}i)\frac{1}{2}(1 + \sqrt{3}i)$$

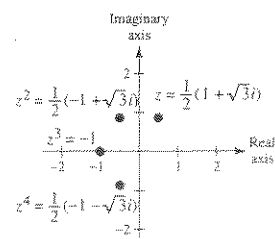
$$= -1$$

$$z^4 = z^3 z$$

$$= (-1)\frac{1}{2}(1 + \sqrt{3}i)$$

$$= \frac{1}{2}(-1 - \sqrt{3}i)$$

The absolute value of each is 1.



$$55. \left[3 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \right] \left[4 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \right] = (3)(4) \left[\cos \left(\frac{\pi}{3} + \frac{\pi}{6} \right) + i \sin \left(\frac{\pi}{3} + \frac{\pi}{6} \right) \right] = 12 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$

$$56. \left[\frac{3}{2} \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \right] \left[6 \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) \right] = \frac{3}{2}(6) \left[\cos \left(\frac{\pi}{6} + \frac{\pi}{4} \right) + i \sin \left(\frac{\pi}{6} + \frac{\pi}{4} \right) \right] \\ = 9 \left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right)$$

$$57. \left[\frac{5}{3} (\cos 140^\circ + i \sin 140^\circ) \right] \left[\frac{2}{3} (\cos 60^\circ + i \sin 60^\circ) \right] = \left(\frac{5}{3} \right) \left(\frac{2}{3} \right) [\cos(140^\circ + 60^\circ) + i \sin(140^\circ + 60^\circ)] \\ = \frac{10}{9} (\cos 200^\circ + i \sin 200^\circ)$$

$$58. \left[\frac{1}{2} (\cos 115^\circ + i \sin 115^\circ) \right] \left[\frac{4}{3} (\cos 300^\circ + i \sin 300^\circ) \right] = \frac{1}{2} \left(\frac{4}{3} \right) [\cos(115^\circ + 300^\circ) + i \sin(115^\circ + 300^\circ)] \\ = \frac{2}{3} (\cos 415^\circ + i \sin 415^\circ) = \frac{2}{3} (\cos 55^\circ + i \sin 55^\circ)$$

$$59. \left[\frac{11}{20} (\cos 290^\circ + i \sin 290^\circ) \right] \left[\frac{2}{5} (\cos 200^\circ + i \sin 200^\circ) \right] = \left(\frac{11}{20} \right) \left(\frac{2}{5} \right) [\cos(290^\circ + 200^\circ) + i \sin(290^\circ + 200^\circ)] \\ = \frac{11}{50} (\cos 490^\circ + i \sin 490^\circ) \\ = \frac{11}{50} (\cos 130^\circ + i \sin 130^\circ)$$

$$60. (\cos 5^\circ + i \sin 5^\circ)(\cos 20^\circ + i \sin 20^\circ) = \cos(5^\circ + 20^\circ) + i \sin(5^\circ + 20^\circ) \\ = \cos 25^\circ + i \sin 25^\circ$$

$$61. \frac{\cos 50^\circ + i \sin 50^\circ}{\cos 20^\circ + i \sin 20^\circ} = \cos(50^\circ - 20^\circ) + i \sin(50^\circ - 20^\circ) = \cos 30^\circ + i \sin 30^\circ$$

$$62. \frac{5[\cos(4.3) + i \sin(4.3)]}{4[\cos(2.1) + i \sin(2.1)]} = \frac{5}{4} [\cos(4.3 - 2.1) + i \sin(4.3 - 2.1)] = \frac{5}{4} [\cos(2.2) + i \sin(2.2)]$$

$$63. \frac{2(\cos 120^\circ + i \sin 120^\circ)}{4(\cos 40^\circ + i \sin 40^\circ)} = \frac{1}{2} [\cos(120^\circ - 40^\circ) + i \sin(120^\circ - 40^\circ)] = \frac{1}{2} (\cos 80^\circ + i \sin 80^\circ)$$

$$64. \frac{\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}}{\cos \pi + i \sin \pi} = \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}$$

$$65. \frac{18(\cos 54^\circ + i \sin 54^\circ)}{3(\cos 102^\circ + i \sin 102^\circ)} = 6(\cos(54^\circ - 102^\circ) + i \sin(54^\circ - 102^\circ)) \\ = 6(\cos(-48^\circ) + i \sin(-48^\circ)) = 6(\cos 312^\circ + i \sin 312^\circ)$$

$$66. \frac{9(\cos 20^\circ + i \sin 20^\circ)}{5(\cos 75^\circ + i \sin 75^\circ)} = \frac{9}{5} [\cos(20^\circ - 75^\circ) + i \sin(20^\circ - 75^\circ)] \\ = \frac{9}{5} [\cos(-55^\circ) + i \sin(-55^\circ)] \\ = \frac{9}{5} [\cos 305^\circ + i \sin 305^\circ]$$

$$67. (a) 2 - 2i = 2\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$$

$$1 + i = \sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$$

$$(b) (2 - 2i)(1 + i) = 2\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) = 4(\cos 2\pi + i \sin 2\pi) = 4$$

$$(c) (2 - 2i)(1 + i) = 2 + 2 = 4$$

$$68. (a) 3 - 3i = 3\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$$

$$1 - i = \sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$$

$$(b) (3 - 3i)(1 - i) = 3\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right) = 6\left(\cos \frac{7\pi}{2} + i \sin \frac{7\pi}{2}\right) = -6i$$

$$(c) (3 - 3i)(1 - i) = 3 - 3 - 6i = -6i$$

$$69. (a) 2 + 2i = 2\sqrt{2}(\cos 45^\circ + i \sin 45^\circ)$$

$$1 - i = \sqrt{2}[\cos(-45^\circ) + i \sin(-45^\circ)]$$

$$(b) (2 + 2i)(1 - i) = [2\sqrt{2}(\cos 45^\circ + i \sin 45^\circ)][\sqrt{2}(\cos(-45^\circ) + i \sin(-45^\circ))] = 4(\cos 0^\circ + i \sin 0^\circ) = 4$$

$$(c) (2 + 2i)(1 - i) = 2 - 2i + 2i - 2i^2 = 2 + 2 = 4$$

$$70. (a) \sqrt{3} + i = 2(\cos 30^\circ + i \sin 30^\circ)$$

$$1 + i = \sqrt{2}(\cos 45^\circ + i \sin 45^\circ)$$

$$(b) (\sqrt{3} + i)(1 + i) = [2(\cos 30^\circ + i \sin 30^\circ)][\sqrt{2}(\cos 45^\circ + i \sin 45^\circ)]$$

$$= 2\sqrt{2}(\cos 75^\circ + i \sin 75^\circ)$$

$$= 2\sqrt{2}\left[\left(\frac{\sqrt{6} - \sqrt{2}}{4}\right) + \left(\frac{\sqrt{6} + \sqrt{2}}{4}\right)i\right]$$

$$= (\sqrt{3} - 1) + (\sqrt{3} + 1)i$$

$$(c) (\sqrt{3} + i)(1 + i) = \sqrt{3} + (\sqrt{3} + 1)i + i^2 = (\sqrt{3} - 1) + (\sqrt{3} + 1)i$$

$$71. (a) -2i = 2[\cos(-90^\circ) + i \sin(-90^\circ)]$$

$$1 + i = \sqrt{2}(\cos 45^\circ + i \sin 45^\circ)$$

$$(b) -2i(1 + i) = 2[\cos(-90^\circ) + i \sin(-90^\circ)][\sqrt{2}(\cos 45^\circ + i \sin 45^\circ)]$$

$$= 2\sqrt{2}[\cos(-45^\circ) + i \sin(-45^\circ)]$$

$$= 2\sqrt{2}\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right] = 2 - 2i$$

$$(c) -2i(1 + i) = -2i - 2i^2 = -2i + 2 = 2 - 2i$$

$$\begin{aligned}
 72. \quad (a) \quad 3i &= 3\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) & (b) \quad 3i(1+i) &= 3\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) \\
 1+i &= \sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) & &= 3\sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) \\
 (c) \quad 3i(1+i) &= 3i - 3 = -3 + 3i & &= 3\sqrt{2}\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = -3 + 3i
 \end{aligned}$$

$$\begin{aligned}
 73. \quad (a) \quad -2i &= 2\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right) \\
 \sqrt{3} - i &= 2\left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right) \\
 (b) \quad -2i(\sqrt{3} - i) &= 2\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)2\left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right) \\
 &= 4\left(\cos \frac{20\pi}{6} + i \sin \frac{20\pi}{6}\right) \\
 &= 4\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \\
 &= -2 - 2\sqrt{3}i \\
 (c) \quad -2i(\sqrt{3} - i) &= -2\sqrt{3}i - 2
 \end{aligned}$$

$$\begin{aligned}
 74. \quad (a) \quad -i &= \cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \\
 1 + \sqrt{3}i &= 2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right) \\
 (b) \quad -i(1 + \sqrt{3}i) &= \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right) \\
 &= 2\left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right) \\
 &= 2\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) = \sqrt{3} - i \\
 (c) \quad (-i)(1 + \sqrt{3}i) &= -i + \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 75. \quad (a) \quad 2 &= 2(\cos 0 + i \sin 0) \\
 1 - i &= \sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right) \\
 (b) \quad 2(1 - i) &= 2\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right) \\
 &= 2\sqrt{2}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) \\
 &= 2(1 - i) = 2 - 2i \\
 (c) \quad 2(1 - i) &= 2 - 2i
 \end{aligned}$$

$$\begin{aligned}
 76. \quad (a) \quad -4 &= 4(\cos \pi + i \sin \pi) \\
 1 + i &= \sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) \\
 (b) \quad -4(1 + i) &= 4\sqrt{2}\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right) \\
 &= 4\sqrt{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) \\
 &= -4 - 4i \\
 (c) \quad -4(1 + i) &= -4 - 4i
 \end{aligned}$$

$$77. (a) \quad 3 + 3i = 3\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$$

$$1 - \sqrt{3}i = 2\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)$$

$$\begin{aligned} (b) \quad \frac{3 + 3i}{1 - \sqrt{3}i} &= \frac{3\sqrt{2}}{2}\left(\cos\left(\frac{\pi}{4} - \frac{5\pi}{3}\right) + i \sin\left(\frac{\pi}{4} - \frac{5\pi}{3}\right)\right) \\ &= \frac{3\sqrt{2}}{2}\left(\cos\left(-\frac{17\pi}{12}\right) + i \sin\left(-\frac{17\pi}{12}\right)\right) \\ &\approx -0.549 + 2.049i \end{aligned}$$

$$\begin{aligned} (c) \quad \frac{3 + 3i}{1 - \sqrt{3}i} \cdot \frac{1 + \sqrt{3}i}{1 + \sqrt{3}i} &= \frac{3 - 3\sqrt{3} + (3 + 3\sqrt{3})i}{4} \\ &\approx -0.549 + 2.049i \end{aligned}$$

$$78. (a) \quad 2 + 2i = 2\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$$

$$1 + \sqrt{3}i = 2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$$

$$\begin{aligned} (b) \quad \frac{2 + 2i}{1 + \sqrt{3}i} &= \frac{2\sqrt{2}}{2}\left(\cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right)\right) \\ &= \sqrt{2}\left(\cos\left(-\frac{\pi}{12}\right) + i \sin\left(-\frac{\pi}{12}\right)\right) \\ &\approx 1.366 - 0.366i \end{aligned}$$

$$\begin{aligned} (c) \quad \frac{2 + 2i}{1 + \sqrt{3}i} \cdot \frac{1 - \sqrt{3}i}{1 - \sqrt{3}i} &= \frac{2 + 2\sqrt{3} + (2 - 2\sqrt{3})i}{4} = \frac{1 + \sqrt{3}}{2} + \frac{1 - \sqrt{3}}{2}i \\ &\approx 1.366 - 0.366i \end{aligned}$$

$$79. (a) \quad 5 = 5(\cos 0 + i \sin 0)$$

$$2 + 2i = 2\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$$

$$\begin{aligned} (b) \quad \frac{5}{2 + 2i} &= \frac{5(\cos 0 + i \sin 0)}{2\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)} \\ &= \frac{5}{2\sqrt{2}}\left(\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right)\right) \\ &= \frac{5}{2\sqrt{2}}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) = \frac{5}{4} - \frac{5}{4}i \end{aligned}$$

$$(c) \quad \frac{5}{2 + 2i} \cdot \frac{2 - 2i}{2 - 2i} = \frac{5(2 - 2i)}{4 + 4} = \frac{5}{4} - \frac{5}{4}i$$

$$80. (a) \quad 2 = 2(\cos 0 + i \sin 0)$$

$$\sqrt{3} - i = 2\left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right)$$

$$\begin{aligned} (b) \quad \frac{2}{\sqrt{3} - i} &= \frac{2}{2}\left(\cos\left(-\frac{11\pi}{6}\right) + i \sin\left(-\frac{11\pi}{6}\right)\right) \\ &= \frac{\sqrt{3}}{2} + \frac{1}{2}i \end{aligned}$$

$$(c) \quad \frac{2}{\sqrt{3} - i} \cdot \frac{\sqrt{3} + i}{\sqrt{3} + i} = \frac{2(\sqrt{3} + i)}{4} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$$

81. (a) $4i = 4\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$

$$-1 + i = \sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$$

$$\begin{aligned} \text{(b)} \quad \frac{4i}{-1+i} &= \frac{4}{\sqrt{2}}\left(\cos\left(\frac{\pi}{2} - \frac{3\pi}{4}\right) + i \sin\left(\frac{\pi}{2} - \frac{3\pi}{4}\right)\right) \\ &= \frac{4}{\sqrt{2}}\left(\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right)\right) \\ &= \frac{4}{\sqrt{2}}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) = 2 - 2i \end{aligned}$$

$$\text{(c)} \quad \frac{4i}{-1+i} \cdot \frac{-1-i}{-1-i} = \frac{-4i+4}{2} = 2-2i$$

82. (a) $2i = 2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$

$$1 - \sqrt{3}i = 2\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)$$

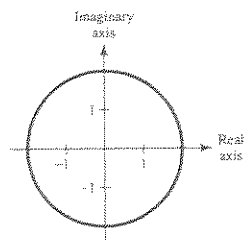
$$\begin{aligned} \text{(b)} \quad \frac{2i}{1-\sqrt{3}i} &= \frac{2}{2}\left(\cos\left(\frac{\pi}{2} - \frac{5\pi}{3}\right) + i \sin\left(\frac{\pi}{2} - \frac{5\pi}{3}\right)\right) \\ &= \cos\left(-\frac{7\pi}{6}\right) + i \sin\left(-\frac{7\pi}{6}\right) = \frac{-\sqrt{3}}{2} + \frac{1}{2}i \end{aligned}$$

$$\text{(c)} \quad \frac{2i}{1-\sqrt{3}i} \cdot \frac{1+\sqrt{3}i}{1+\sqrt{3}i} = \frac{2i-2\sqrt{3}}{4} = \frac{-\sqrt{3}}{2} + \frac{1}{2}i$$

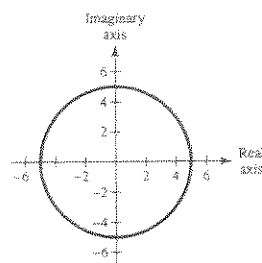
83. Let $z = x + iy$ such that:

$$|z| = 2 \Rightarrow 2 = \sqrt{x^2 + y^2} \Rightarrow 4 = x^2 + y^2$$

Circle with radius of 2



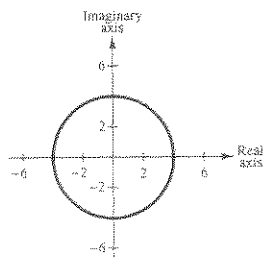
84. $|z| = 5$, Circle of radius 5



85. Let $z = x + iy$.

$$|z| = 4 \Rightarrow 4 = \sqrt{x^2 + y^2} \Rightarrow x^2 + y^2 = 16$$

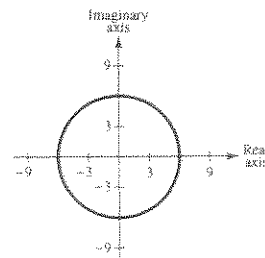
Circle of radius 4



86. Let $z = x + iy$.

$$|z| = 6 \Rightarrow 6 = \sqrt{x^2 + y^2} \Rightarrow x^2 + y^2 = 36$$

Circle of radius 6



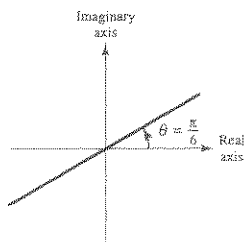
87. $\theta = \frac{\pi}{6}$

 Let $z = x + iy$ such that:

$$\tan \frac{\pi}{6} = \frac{y}{x} \Rightarrow$$

$$\frac{y}{x} = \frac{1}{\sqrt{3}} \Rightarrow y = \frac{1}{\sqrt{3}}x$$

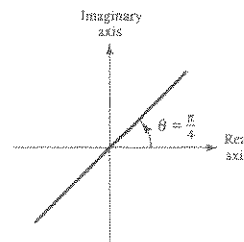
Line



88. $\theta = \frac{\pi}{4} = \arctan 1$

Line

$$y = x$$



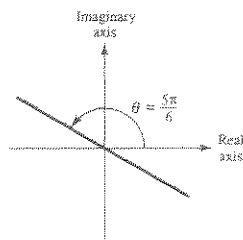
89. $\theta = \frac{5\pi}{6}$

 Let $z = x + iy$ such that:

$$\tan \frac{5\pi}{6} = \frac{y}{x} \Rightarrow$$

$$\frac{y}{x} = -\frac{\sqrt{3}}{3} \Rightarrow y = -\frac{\sqrt{3}}{3}x$$

Line



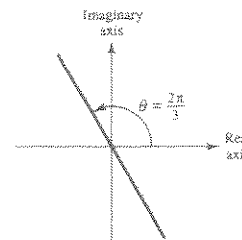
90. $\theta = \frac{2\pi}{3}$

 Let $z = x + iy$ such that:

$$\tan \frac{2\pi}{3} = \frac{y}{x} \Rightarrow$$

$$-\sqrt{3} = \frac{y}{x} \Rightarrow y = -\sqrt{3}x$$

Line



$$\begin{aligned} 91. (1 + i)^3 &= \left[\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) \right]^3 \\ &= (\sqrt{2})^3 \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) \\ &= 2\sqrt{2} \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right) \\ &= -2 + 2i \end{aligned}$$

$$\begin{aligned} 92. (2 + 2i)^6 &= \left[2\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) \right]^6 \\ &= (2\sqrt{2})^6 \left(\cos \frac{6\pi}{4} + i \sin \frac{6\pi}{4} \right) \\ &= 512 \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right) \\ &= -512i \end{aligned}$$

$$\begin{aligned} 93. (-1 + i)^{10} &= \left[\sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) \right]^{10} \\ &= (\sqrt{2})^{10} \left(\cos \frac{30\pi}{4} + i \sin \frac{30\pi}{4} \right) \\ &= 32 \left[\cos \left(\frac{3\pi}{2} + 6\pi \right) + i \sin \left(\frac{3\pi}{2} + 6\pi \right) \right] \\ &= 32 \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right) \\ &= 32[0 + i(-1)] = -32i \end{aligned}$$

$$\begin{aligned} 94. (1 - i)^8 &= \left[\sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right) \right]^8 \\ &= (\sqrt{2})^8 (\cos 14\pi + i \sin 14\pi) \\ &= 16 \end{aligned}$$

$$\begin{aligned} 95. 2(\sqrt{3} + i)^5 &= 2 \left[2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \right]^5 \\ &= 2 \left[2^5 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right) \right] \\ &= 64 \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) \\ &= -32\sqrt{3} + 32i \end{aligned}$$

$$\begin{aligned} 96. 4(1 - \sqrt{3}i)^3 &= 4 \left[2 \left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right) \right]^3 \\ &= 4[2^3 (\cos 5\pi + i \sin 5\pi)] \\ &= 32(-1) \\ &= -32 \end{aligned}$$

$$97. [5(\cos 20^\circ + i \sin 20^\circ)]^3 = 5^3(\cos 60^\circ + i \sin 60^\circ) = \frac{125}{2} + \frac{125\sqrt{3}}{2}i$$

$$\begin{aligned} 98. [3(\cos 150^\circ + i \sin 150^\circ)]^4 &= 3^4(\cos 600^\circ + i \sin 600^\circ) \\ &= 81(\cos 240^\circ + i \sin 240^\circ) \\ &= 81(-\cos 60^\circ - i \sin 60^\circ) \\ &= -\frac{81}{2} - \frac{81\sqrt{3}}{2}i \end{aligned}$$

$$\begin{aligned} 99. \left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right)^{10} &= \cos \frac{25\pi}{2} + i \sin \frac{25\pi}{2} \\ &= \cos\left(12\pi + \frac{\pi}{2}\right) + i \sin\left(12\pi + \frac{\pi}{2}\right) = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} = i \end{aligned}$$

$$100. \left[2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)\right]^{12} = 2^{12}(\cos 6\pi + i \sin 6\pi) = 2^{12} = 4096$$

$$\begin{aligned} 101. [2(\cos 1.25 + i \sin 1.25)]^4 &= 2^4(\cos 5 + i \sin 5) \\ &\approx 4.5386 - 15.3428i \end{aligned}$$

$$\begin{aligned} 102. [4(\cos 2.8 + i \sin 2.8)]^5 &= 4^5(\cos 14 + i \sin 14) \\ &\approx 140.02 + 1014.38i \end{aligned}$$

$$\begin{aligned} 103. [2(\cos \pi + i \sin \pi)]^8 &= 2^8(\cos 8\pi + i \sin 8\pi) \\ &= 256(1) = 256 \end{aligned}$$

$$\begin{aligned} 104. (\cos 0 + i \sin 0)^{20} &= \cos 0 + i \sin 0 \\ &= 1 \end{aligned}$$

$$105. (3 - 2i)^5 = -597 - 122i$$

$$\begin{aligned} 106. (\sqrt{5} - 4i)^4 &= [\sqrt{21}(\cos(5.2221) + i \sin(5.2221))]^4 \\ &= 441[\cos(20.8884) + i \sin(20.8884)] \\ &= -199 + 393.55i \end{aligned}$$

$$\begin{aligned} 107. [4(\cos 10^\circ + i \sin 10^\circ)]^6 &= 4^6(\cos 60^\circ + i \sin 60^\circ) \\ &= 4096\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\ &= 2048 + 2048\sqrt{3}i \end{aligned}$$

$$\begin{aligned} 108. [3(\cos 15^\circ + i \sin 15^\circ)]^4 &= 81(\cos 60^\circ + i \sin 60^\circ) \\ &= \frac{81}{2} + \frac{81\sqrt{3}}{2}i \end{aligned}$$

$$\begin{aligned} 109. \left[3\left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8}\right)\right]^2 &= 3^2\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) \\ &= 9\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) \\ &= \frac{9}{2}\sqrt{2} + \frac{9}{2}\sqrt{2}i \end{aligned}$$

$$\begin{aligned} 110. \left[2\left(\cos \frac{\pi}{10} + i \sin \frac{\pi}{10}\right)\right]^5 &= 32\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) \\ &= 32i \end{aligned}$$

$$\begin{aligned} 111. \left[-\frac{1}{2}(1 + \sqrt{3}i)\right]^6 &= \left[\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right]^6 \\ &= \cos 8\pi + i \sin 8\pi \\ &= 1 \end{aligned}$$

112. $2^{-1/4}(1 - i)$ is a fourth root of -2 if
 $-2 = [2^{-1/4}(1 - i)]^4$.

$$\begin{aligned} [2^{-1/4}(1 - i)]^4 &= (2^{-1/4})^4(1 - i)^4 \\ &= 2^{-1}(1 - i)^4 \\ &= \frac{1}{2}(1 - i)^2(1 - i)^2 \\ &= \frac{1}{2}(-2i)(-2i) \\ &= \frac{1}{2}(4i^2) \\ &= \frac{1}{2}(-4) = -2 \end{aligned}$$

114. (a) In trigonometric form we have:

$$\begin{aligned} &3(\cos 45^\circ + i \sin 45^\circ) \\ &3(\cos 135^\circ + i \sin 135^\circ) \\ &3(\cos 225^\circ + i \sin 225^\circ) \\ &3(\cos 315^\circ + i \sin 315^\circ) \end{aligned}$$

- (b) There are four roots evenly spaced around a circle of radius 3. Therefore, they represent the fourth roots of some number of modulus 81. Raising them to the fourth power shows that they are all fourth roots of -81 .

$$\begin{aligned} \text{(c)} \quad &[3(\cos 45^\circ + i \sin 45^\circ)]^4 = -81 \\ &[3(\cos 135^\circ + i \sin 135^\circ)]^4 = -81 \\ &[3(\cos 225^\circ + i \sin 225^\circ)]^4 = -81 \\ &[3(\cos 315^\circ + i \sin 315^\circ)]^4 = -81 \end{aligned}$$

116. (a) In trigonometric form we have:

$$\begin{aligned} &2(\cos 30^\circ + i \sin 30^\circ) \\ &2(\cos 90^\circ + i \sin 90^\circ) \\ &2(\cos 150^\circ + i \sin 150^\circ) \\ &2(\cos 210^\circ + i \sin 210^\circ) \\ &2(\cos 270^\circ + i \sin 270^\circ) \\ &2(\cos 330^\circ + i \sin 330^\circ) \end{aligned}$$

- (b) There are six roots evenly spaced around a circle of radius 2. Raising them to the sixth power shows that they are the six sixth roots of -64 .

113. (a) In trigonometric form we have:

$$\begin{aligned} &2(\cos 30^\circ + i \sin 30^\circ) \\ &2(\cos 150^\circ + i \sin 150^\circ) \\ &2(\cos 270^\circ + i \sin 270^\circ) \end{aligned}$$

- (b) There are three roots evenly spaced around a circle of radius 2. Therefore, they represent the cube roots of some number of modulus 8. Cubing them shows that they are all cube roots of $8i$.

$$\begin{aligned} \text{(c)} \quad &[2(\cos 30^\circ + i \sin 30^\circ)]^3 = 8i \\ &[2(\cos 150^\circ + i \sin 150^\circ)]^3 = 8i \\ &[2(\cos 270^\circ + i \sin 270^\circ)]^3 = 8i \end{aligned}$$

115. (a) In trigonometric form we have:

$$\begin{aligned} &\cos 120^\circ + i \sin 120^\circ \\ &\cos 240^\circ + i \sin 240^\circ \\ &\cos 0^\circ + i \sin 0^\circ \end{aligned}$$

- (b) These are the three cube roots of 1.

$$\begin{aligned} \text{(c)} \quad &(\cos 120^\circ + i \sin 120^\circ)^3 = 1 \\ &(\cos 240^\circ + i \sin 240^\circ)^3 = 1 \\ &(\cos 0^\circ + i \sin 0^\circ)^3 = 1 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad &[2(\cos 30^\circ + i \sin 30^\circ)]^6 = -64 \\ &[2(\cos 90^\circ + i \sin 90^\circ)]^6 = -64 \\ &[2(\cos 150^\circ + i \sin 150^\circ)]^6 = -64 \\ &[2(\cos 210^\circ + i \sin 210^\circ)]^6 = -64 \\ &[2(\cos 270^\circ + i \sin 270^\circ)]^6 = -64 \\ &[2(\cos 330^\circ + i \sin 330^\circ)]^6 = -64 \end{aligned}$$

$$117. 2i = 2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$$

Square roots:

$$\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) = 1 + i$$

$$\sqrt{2}\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right) = -1 - i$$

$$119. -3i = 3\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$$

Square roots:

$$\sqrt{3}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) = -\frac{\sqrt{6}}{2} + \frac{\sqrt{6}}{2}i$$

$$\sqrt{3}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right) = \frac{\sqrt{6}}{2} - \frac{\sqrt{6}}{2}i$$

$$121. 2 - 2i = 2\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$$

Square roots:

$$8^{1/4}\left(\cos \frac{7\pi}{8} + i \sin \frac{7\pi}{8}\right) \approx -1.554 + 0.644i$$

$$8^{1/4}\left(\cos \frac{15\pi}{8} + i \sin \frac{15\pi}{8}\right) \approx 1.554 - 0.644i$$

$$123. 1 + \sqrt{3}i = 2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$$

Square roots:

$$\sqrt{2}\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right) = \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$$

$$\sqrt{2}\left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}\right) = -\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i$$

$$125. (a) \text{ Square roots of } 5(\cos 120^\circ + i \sin 120^\circ):$$

$$\sqrt{5}\left[\cos\left(\frac{120^\circ + 360^\circ k}{2}\right) + i \sin\left(\frac{120^\circ + 360^\circ k}{2}\right)\right], k = 0, 1$$

$$\sqrt{5}(\cos 60^\circ + i \sin 60^\circ)$$

$$\sqrt{5}(\cos 240^\circ + i \sin 240^\circ)$$

$$(c) \frac{\sqrt{5}}{2} + \frac{\sqrt{15}}{2}i, -\frac{\sqrt{5}}{2} - \frac{\sqrt{15}}{2}i$$

$$118. 5i = 5\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$$

Square roots:

$$\sqrt{5}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) = \frac{\sqrt{10}}{2} + \frac{\sqrt{10}}{2}i$$

$$\sqrt{5}\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right) = -\frac{\sqrt{10}}{2} - \frac{\sqrt{10}}{2}i$$

$$120. -6i = 6\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$$

Square roots:

$$\sqrt{6}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) = -\sqrt{3} + \sqrt{3}i$$

$$\sqrt{6}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right) = \sqrt{3} - \sqrt{3}i$$

$$122. 2 + 2i = 2\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$$

Square roots:

$$8^{1/4}\left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8}\right) \approx 1.554 + 0.644i$$

$$8^{1/4}\left(\cos \frac{9\pi}{8} + i \sin \frac{9\pi}{8}\right) \approx -1.554 - 0.644i$$

$$124. 1 - \sqrt{3}i = 2\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)$$

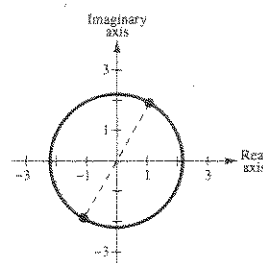
Square roots:

$$\sqrt{2}\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right) = \sqrt{2}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$$

$$= -\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$$

$$\sqrt{2}\left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right) = \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i$$

(b)



126. (a) Square roots of $16(\cos 60^\circ + i \sin 60^\circ)$:

$$\sqrt[4]{16} \left[\cos \left(\frac{60^\circ + 360^\circ k}{2} \right) + i \sin \left(\frac{60^\circ + 360^\circ k}{2} \right) \right], k = 0, 1$$

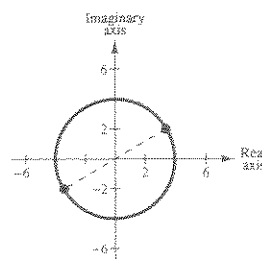
$$4(\cos 30^\circ + i \sin 30^\circ)$$

$$4(\cos 210^\circ + i \sin 210^\circ)$$

$$(c) 4 \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = 2\sqrt{3} + 2i$$

$$4 \left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i \right) = -2\sqrt{3} - 2i$$

(b)


 127. (a) Fourth roots of $16 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right)$:

$$\sqrt[4]{16} \left[\cos \left(\frac{(4\pi/3) + 2k\pi}{4} \right) + i \sin \left(\frac{(4\pi/3) + 2k\pi}{4} \right) \right], k = 0, 1, 2, 3$$

$$2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

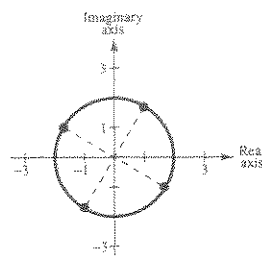
$$2 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$$

$$2 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right)$$

$$2 \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right)$$

$$(c) 1 + \sqrt{3}i, -\sqrt{3} + i, -1 - \sqrt{3}i, \sqrt{3} - i$$

(b)


 128. (a) Fifth roots of $32 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$:

$$\sqrt[5]{32} \left[\cos \left(\frac{(5\pi/6) + 2k\pi}{5} \right) + i \sin \left(\frac{(5\pi/6) + 2k\pi}{5} \right) \right]$$

$$k = 0, 1, 2, 3, 4$$

$$k = 0: 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

$$k = 1: 2 \left(\cos \frac{17\pi}{30} + i \sin \frac{17\pi}{30} \right)$$

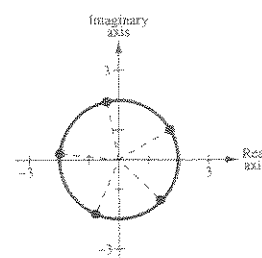
$$k = 2: 2 \left(\cos \frac{29\pi}{30} + i \sin \frac{29\pi}{30} \right)$$

$$k = 3: 2 \left(\cos \frac{41\pi}{30} + i \sin \frac{41\pi}{30} \right)$$

$$k = 4: 2 \left(\cos \frac{53\pi}{30} + i \sin \frac{53\pi}{30} \right)$$

$$(c) 1.732 + i, -0.4158 + 1.956i, -1.989 + 0.2091i, -0.8135 - 1.827i, 1.486 - 1.338i$$

(b)



129. (a) Cube roots of $-27i = 27\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$:

$$(27)^{1/3} \left[\cos \left(\frac{(3\pi/2) + 2k\pi}{3} \right) + i \sin \left(\frac{(3\pi/2) + 2k\pi}{3} \right) \right], \quad k = 0, 1, 2$$

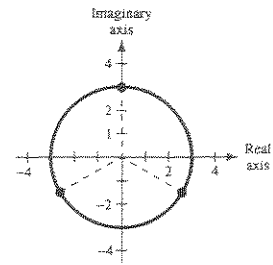
$$3 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$

$$3 \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right)$$

$$3 \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right)$$

(c) $3i, -\frac{3\sqrt{3}}{2} - \frac{3}{2}i, \frac{3\sqrt{3}}{2} - \frac{3}{2}i$

(b)



130. (a) Fourth roots of $625i = 625\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$:

$$\sqrt[4]{625} \left[\cos \left(\frac{(\pi/2) + 2k\pi}{4} \right) + i \sin \left(\frac{(\pi/2) + 2k\pi}{4} \right) \right]$$

$$k = 0, 1, 2, 3$$

$$k = 0: 5 \left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8} \right)$$

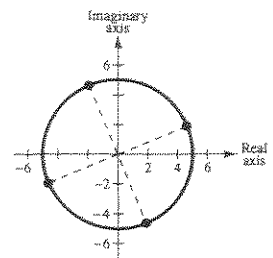
$$k = 1: 5 \left(\cos \frac{5\pi}{8} + i \sin \frac{5\pi}{8} \right)$$

$$k = 2: 5 \left(\cos \frac{9\pi}{8} + i \sin \frac{9\pi}{8} \right)$$

$$k = 3: 5 \left(\cos \frac{13\pi}{8} + i \sin \frac{13\pi}{8} \right)$$

(c) $4.619 + 1.913i, -1.913 + 4.619i, -4.619 - 1.913i, 1.913 - 4.619i$

(b)



131. (a) Cube roots of $-\frac{125}{2}(1 + \sqrt{3}i) = 125\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$:

$$\sqrt[3]{125} \left[\cos \left(\frac{(4\pi/3) + 2k\pi}{3} \right) + i \sin \left(\frac{(4\pi/3) + 2k\pi}{3} \right) \right], \quad k = 0, 1, 2$$

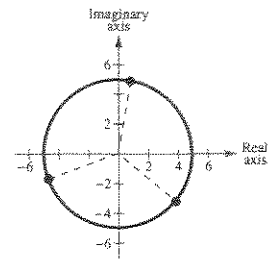
$$5 \left(\cos \frac{4\pi}{9} + i \sin \frac{4\pi}{9} \right)$$

$$5 \left(\cos \frac{10\pi}{9} + i \sin \frac{10\pi}{9} \right)$$

$$5 \left(\cos \frac{16\pi}{9} + i \sin \frac{16\pi}{9} \right)$$

(c) $0.8682 + 4.9240i, -4.6985 - 1.7101i, 3.8302 - 3.2139i$

(b)



132. (a) Cube roots of $-4\sqrt{2}(1-i) = 8\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$:

$$\sqrt[3]{8}\left[\cos\left(\frac{(3\pi/4) + 2k\pi}{3}\right) + i \sin\left(\frac{(3\pi/4) + 2k\pi}{3}\right)\right]$$

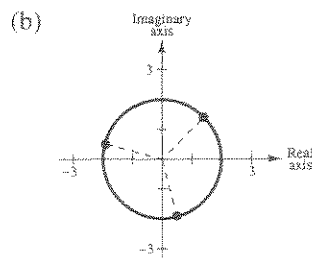
$$k = 0, 1, 2$$

$$k = 0: 2\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$$

$$k = 1: 2\left(\cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12}\right)$$

$$k = 2: 2\left(\cos \frac{19\pi}{12} + i \sin \frac{19\pi}{12}\right)$$

(c) $1.414 + 1.414i, -1.932 + 0.5176i, 0.5176 - 1.9319i$



133. (a) Cube roots of $64i = 64\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$:

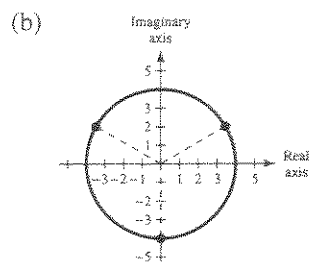
$$(64)^{1/3}\left[\cos\left(\frac{(\pi/2) + 2k\pi}{3}\right) + i \sin\left(\frac{(\pi/2) + 2k\pi}{3}\right)\right], \quad k = 0, 1, 2$$

$$4\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$$

$$4\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right)$$

$$4\left(\cos \frac{9\pi}{6} + i \sin \frac{9\pi}{6}\right) = 4\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$$

(c) $2\sqrt{3} + 2i, -2\sqrt{3} + 2i, -4i$



134. (a) Fourth roots of $i = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$:

$$\sqrt[4]{1}\left[\cos\left(\frac{(\pi/2) + 2k\pi}{4}\right) + i \sin\left(\frac{(\pi/2) + 2k\pi}{4}\right)\right]$$

$$k = 0, 1, 2, 3$$

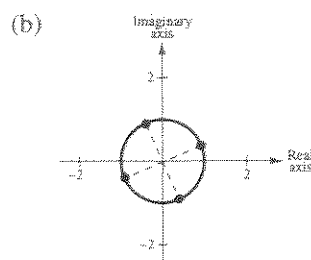
$$k = 0: \cos \frac{\pi}{8} + i \sin \frac{\pi}{8}$$

$$k = 1: \cos \frac{5\pi}{8} + i \sin \frac{5\pi}{8}$$

$$k = 2: \cos \frac{9\pi}{8} + i \sin \frac{9\pi}{8}$$

$$k = 3: \cos \frac{13\pi}{8} + i \sin \frac{13\pi}{8}$$

(c) $0.9239 + 0.3827i, -0.3827 + 0.9239i, -0.9239 - 0.3827i, 0.3827 - 0.9239i$



135. (a) Fifth roots of
- $1 = \cos 0 + i \sin 0$
- :

$$\cos \frac{2k\pi}{5} + i \sin \frac{2k\pi}{5}, \quad k = 0, 1, 2, 3, 4$$

$$\cos 0 + i \sin 0$$

$$\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$$

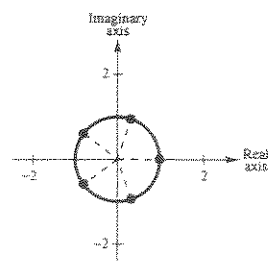
$$\cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}$$

$$\cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5}$$

$$\cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5}$$

- (c)
- $1, 0.3090 + 0.9511i, -0.8090 + 0.5878i, -0.8090 - 0.5878i, 0.3090 - 0.9511i$

(b)



136. (a) Cube roots of
- $1000 = 1000(\cos 0 + i \sin 0)$
- :

$$\sqrt[3]{1000} \left(\cos \frac{2k\pi}{3} + i \sin \frac{2k\pi}{3} \right)$$

$$k = 0, 1, 2$$

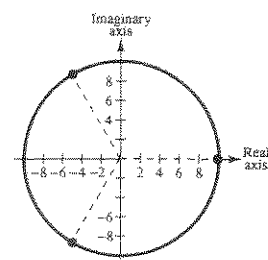
$$k = 0: 10(\cos 0 + i \sin 0) = 10$$

$$k = 1: 10 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

$$k = 2: 10 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right)$$

- (c)
- $10, -5 + 5\sqrt{3}i, -5 - 5\sqrt{3}i$

(b)



137. (a) Cube roots of
- $-125 = 125(\cos 180^\circ + i \sin 180^\circ)$
- are:

$$\sqrt[3]{125} \left[\cos \left(\frac{180 + 360k}{3} \right) + i \sin \left(\frac{180 + 360k}{3} \right) \right], \quad k = 0, 1, 2$$

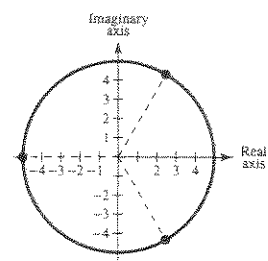
$$5(\cos 60^\circ + i \sin 60^\circ)$$

$$5(\cos 180^\circ + i \sin 180^\circ)$$

$$5(\cos 300^\circ + i \sin 300^\circ)$$

- (c)
- $\frac{5}{2} + \frac{5\sqrt{3}}{2}i, -5, \frac{5}{2} - \frac{5\sqrt{3}}{2}i$

(b)



138. (a) Fourth roots of
- $-4 = 4(\cos 180^\circ + i \sin 180^\circ)$
- :

$$\sqrt[4]{4}(\cos 45^\circ + i \sin 45^\circ)$$

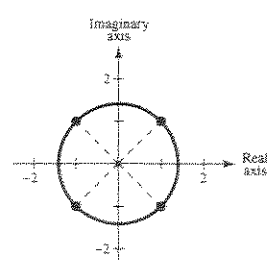
$$\sqrt[4]{4}(\cos 135^\circ + i \sin 135^\circ)$$

$$\sqrt[4]{4}(\cos 225^\circ + i \sin 225^\circ)$$

$$\sqrt[4]{4}(\cos 315^\circ + i \sin 315^\circ)$$

- (c)
- $1 + i, -1 + i, -1 - i, 1 - i$

(b)



139. (a) Fifth roots of $128(-1 + i) = 128\sqrt{2}(\cos 135^\circ + i \sin 135^\circ)$ are:

$$2\sqrt{2}(\cos 27^\circ + i \sin 27^\circ) = 2\sqrt{2}\left(\cos \frac{3\pi}{20} + i \sin \frac{3\pi}{20}\right)$$

$$2\sqrt{2}(\cos 99^\circ + i \sin 99^\circ)$$

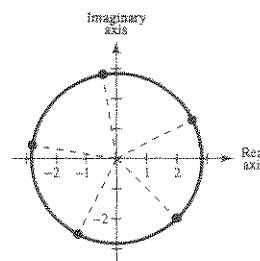
$$2\sqrt{2}(\cos 171^\circ + i \sin 171^\circ)$$

$$2\sqrt{2}(\cos 243^\circ + i \sin 243^\circ)$$

$$2\sqrt{2}(\cos 315^\circ + i \sin 315^\circ)$$

(c) $2.52 + 1.28i, -0.44 + 2.79i, -2.79 + 0.44i, -1.28 - 2.52i, 2 - 2i$

(b)



140. (a) Sixth roots of $729i = 729\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$:

$$\sqrt[6]{729}\left[\cos\left(\frac{\pi/2 + 2k\pi}{6}\right) + i \sin\left(\frac{\pi/2 + 2k\pi}{6}\right)\right], k = 0, 1, 2, 3, 4, 5$$

$$3\left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}\right)$$

$$3\left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12}\right)$$

$$3\left(\cos \frac{9\pi}{12} + i \sin \frac{9\pi}{12}\right)$$

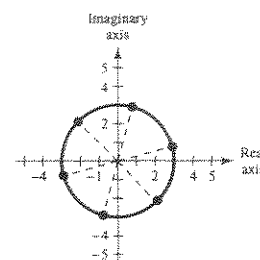
$$3\left(\cos \frac{13\pi}{12} + i \sin \frac{13\pi}{12}\right)$$

$$3\left(\cos \frac{17\pi}{12} + i \sin \frac{17\pi}{12}\right)$$

$$3\left(\cos \frac{21\pi}{12} + i \sin \frac{21\pi}{12}\right)$$

(c) $2.898 + 0.776i, 0.776 + 2.898i, -2.121 + 2.121i, -2.898 - 0.776i, -0.776 - 2.898i, 2.121 - 2.121i$

(b)



141. $x^4 - i = 0$

$$x^4 = i$$

The solutions are the fourth roots of $i = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$:

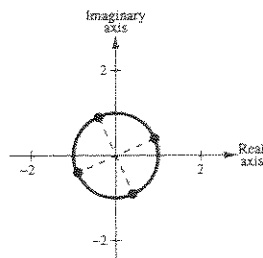
$$\sqrt[4]{1}\left[\cos\left(\frac{(\pi/2) + 2k\pi}{4}\right) + i \sin\left(\frac{(\pi/2) + 2k\pi}{4}\right)\right], k = 0, 1, 2, 3$$

$$\cos \frac{\pi}{8} + i \sin \frac{\pi}{8}$$

$$\cos \frac{5\pi}{8} + i \sin \frac{5\pi}{8}$$

$$\cos \frac{9\pi}{8} + i \sin \frac{9\pi}{8}$$

$$\cos \frac{13\pi}{8} + i \sin \frac{13\pi}{8}$$



142. $x^3 + 27 = 0$

$$x^3 = -27$$

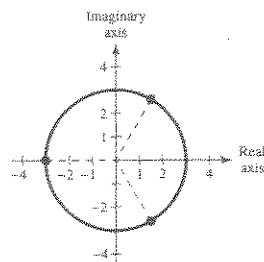
Solutions are cube roots of $-27 = 27(\cos \pi + i \sin \pi)$:

$$3\left(\cos \frac{\pi + 2\pi k}{3} + i \sin \frac{\pi + 2\pi k}{3}\right), k = 0, 1, 2$$

$$3\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right) = \frac{3}{2} + \frac{3\sqrt{3}}{2}i$$

$$3(\cos \pi + i \sin \pi) = -3$$

$$3\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right) = \frac{3}{2} - \frac{3\sqrt{3}}{2}i$$



143. $x^5 = -243$

The solutions are the fifth roots of $-243 = 243[\cos \pi + i \sin \pi]$:

$$\sqrt[5]{243}\left[\cos\left(\frac{\pi + 2k\pi}{5}\right) + i \sin\left(\frac{\pi + 2k\pi}{5}\right)\right], k = 0, 1, 2, 3, 4$$

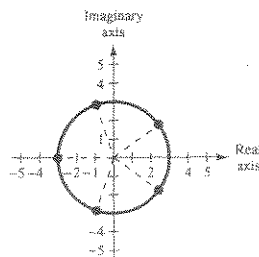
$$3\left(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5}\right)$$

$$3\left(\cos \frac{3\pi}{5} + i \sin \frac{3\pi}{5}\right)$$

$$3\left(\cos \frac{5\pi}{5} + i \sin \frac{5\pi}{5}\right) = -3$$

$$3\left(\cos \frac{7\pi}{5} + i \sin \frac{7\pi}{5}\right)$$

$$3\left(\cos \frac{9\pi}{5} + i \sin \frac{9\pi}{5}\right)$$



144. $x^4 = 81 = 81(\cos 0 + i \sin 0)$

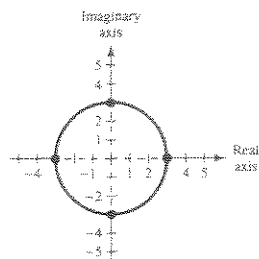
$$3\left[\cos \frac{2\pi k}{4} + i \sin \frac{2\pi k}{4}\right], k = 0, 1, 2, 3$$

$$3(\cos 0 + i \sin 0) = 3$$

$$3\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) = 3i$$

$$3(\cos \pi + i \sin \pi) = -3$$

$$3\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right) = -3i$$



145. $x^4 = -16i$

The solutions are the fourth roots of $-16i = 16\left[\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right]$:

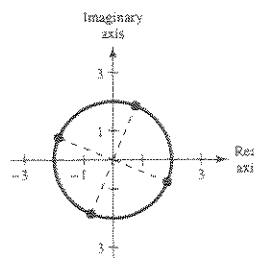
$$\sqrt[4]{16}\left[\cos\left(\frac{(3\pi/2) + 2k\pi}{4}\right) + i \sin\left(\frac{(3\pi/2) + 2k\pi}{4}\right)\right], \quad k = 0, 1, 2, 3$$

$$2\left[\cos\left(\frac{3\pi}{8}\right) + i \sin\left(\frac{3\pi}{8}\right)\right]$$

$$2\left[\cos\left(\frac{7\pi}{8}\right) + i \sin\left(\frac{7\pi}{8}\right)\right]$$

$$2\left[\cos\left(\frac{11\pi}{8}\right) + i \sin\left(\frac{11\pi}{8}\right)\right]$$

$$2\left[\cos\left(\frac{15\pi}{8}\right) + i \sin\left(\frac{15\pi}{8}\right)\right]$$



146. $x^6 - 64i = 0$

$$x^6 = 64i$$

The solutions are the sixth roots of $64i$:

$$\sqrt[6]{64}\left[\cos\left(\frac{(\pi/2) + 2k\pi}{6}\right) + i \sin\left(\frac{(\pi/2) + 2k\pi}{6}\right)\right], \quad k = 0, 1, 2, 3, 4, 5$$

$$k = 0: 2\left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}\right) \approx 1.932 + 0.5176i$$

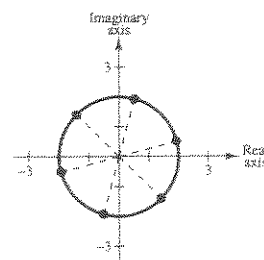
$$k = 1: 2\left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12}\right) \approx 0.5176 + 1.932i$$

$$k = 2: 2\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) \approx -1.414 + 1.414i$$

$$k = 3: 2\left(\cos \frac{13\pi}{12} + i \sin \frac{13\pi}{12}\right) \approx -1.932 - 0.5176i$$

$$k = 4: 2\left(\cos \frac{17\pi}{12} + i \sin \frac{17\pi}{12}\right) \approx -0.5176 - 1.932i$$

$$k = 5: 2\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right) \approx 1.414 - 1.414i$$



147. $x^3 - (1 - i) = 0$

$$x^3 = 1 - i = \sqrt{2}(\cos 315^\circ + i \sin 315^\circ)$$

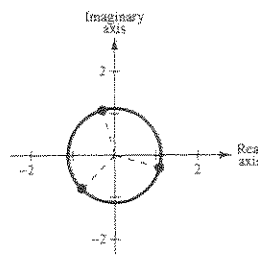
The solutions are the cube roots of $1 - i$:

$$\sqrt[3]{\sqrt{2}}\left[\cos\left(\frac{315^\circ + 360^\circ k}{3}\right) + i \sin\left(\frac{315^\circ + 360^\circ k}{3}\right)\right], \quad k = 0, 1, 2$$

$$\sqrt[3]{2}(\cos 105^\circ + i \sin 105^\circ)$$

$$\sqrt[3]{2}(\cos 225^\circ + i \sin 225^\circ)$$

$$\sqrt[3]{2}(\cos 345^\circ + i \sin 345^\circ)$$



148. $x^4 + (1 + i) = 0$

$$x^4 = -1 - i = \sqrt{2}(\cos 225^\circ + i \sin 225^\circ)$$

The solutions are the fourth roots of $-1 - i$:

$$\sqrt[4]{\sqrt{2}} \left[\cos \left(\frac{225^\circ + 360^\circ k}{4} \right) + i \sin \left(\frac{225^\circ + 360^\circ k}{4} \right) \right]$$

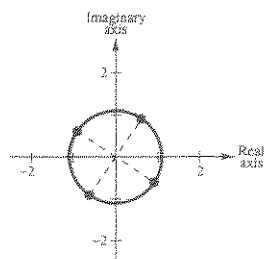
$$k = 0, 1, 2, 3$$

$$k = 0: \sqrt[8]{2}(\cos 56.25^\circ + i \sin 56.25^\circ) \approx 0.6059 + 0.9067i$$

$$k = 1: \sqrt[8]{2}(\cos 146.25^\circ + i \sin 146.25^\circ) \approx -0.9067 + 0.6059i$$

$$k = 2: \sqrt[8]{2}(\cos 236.25^\circ + i \sin 236.25^\circ) \approx -0.6059 - 0.9067i$$

$$k = 3: \sqrt[8]{2}(\cos 326.25^\circ + i \sin 326.25^\circ) \approx 0.9067 - 0.6059i$$



149. $E = I \cdot Z$

$$= (10 + 2i)(4 + 3i)$$

$$= (40 - 6) + (30 + 8)i$$

$$= 34 + 38i$$

150. $E = I \cdot Z$

$$= (12 + 2i)(3 + 5i)$$

$$= (36 - 10) + (60 + 6)i$$

$$= 26 + 66i$$

151. $Z = \frac{E}{I}$

$$= \frac{5 + 5i}{2 + 4i} \cdot \frac{2 - 4i}{2 - 4i}$$

$$= \frac{(10 + 20) + (10 - 20)i}{4 + 16}$$

$$= \frac{30 - 10i}{20}$$

$$= \frac{3}{2} - \frac{1}{2}i$$

152. $Z = \frac{E}{I}$

$$= \frac{4 + 5i}{10 + 2i} \cdot \frac{10 - 2i}{10 - 2i}$$

$$= \frac{(40 + 10) + (50 - 8)i}{100 + 4}$$

$$= \frac{50 + 42i}{104}$$

$$= \frac{25}{52} + \frac{21}{52}i$$

153. $I = \frac{E}{Z}$

$$= \frac{12 + 24i}{12 + 20i} \cdot \frac{12 - 20i}{12 - 20i}$$

$$= \frac{(144 + 480) + (288 - 240)i}{144 + 400}$$

$$= \frac{624 + 48i}{544}$$

$$= \frac{39}{34} + \frac{3}{34}i$$

154. $I = \frac{E}{Z}$

$$= \frac{15 + 12i}{25 + 24i} \cdot \frac{25 - 24i}{25 - 24i}$$

$$= \frac{(375 + 288) + (300 - 360)i}{625 + 576}$$

$$= \frac{663 - 60i}{1201}$$

$$= \frac{663}{1201} - \frac{60}{1201}i$$

155. True. $\left[\frac{1}{2}(1 - \sqrt{3}i) \right]^9 = \left[\frac{1}{2} - \frac{\sqrt{3}}{2}i \right]^9 = -1$

156. False. $(\sqrt{3} + i)^2 = 2 + 2\sqrt{3}i \neq 8i$

157. True

$$\begin{aligned}
 158. \quad \frac{z_1}{z_2} &= \frac{r_1(\cos \theta_1 + i \sin \theta_1)}{r_2(\cos \theta_2 + i \sin \theta_2)} \cdot \frac{\cos \theta_2 - i \sin \theta_2}{\cos \theta_2 - i \sin \theta_2} \\
 &= \frac{r_1}{r_2(\cos^2 \theta_2 + \sin^2 \theta_2)} [\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 + i(\sin \theta_1 \cos \theta_2 - \sin \theta_2 \cos \theta_1)] \\
 &= \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)]
 \end{aligned}$$

$$\begin{aligned}
 159. \quad z &= r(\cos \theta + i \sin \theta) \\
 \bar{z} &= r(\cos \theta - i \sin \theta) \\
 &= r(\cos(-\theta) + i \sin(-\theta))
 \end{aligned}$$

$$\begin{aligned}
 160. \quad (a) \quad z\bar{z} &= [r(\cos \theta + i \sin \theta)][r(\cos(-\theta) + i \sin(-\theta))] \\
 &= r^2[\cos(\theta - \theta) + i \sin(\theta - \theta)] \\
 &= r^2[\cos 0 + i \sin 0] \\
 &= r^2
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad \frac{z}{\bar{z}} &= \frac{r(\cos \theta + i \sin \theta)}{r[\cos(-\theta) + i \sin(-\theta)]} \\
 &= \frac{r}{r} [\cos(\theta - (-\theta)) + i \sin(\theta - (-\theta))] \\
 &= \cos 2\theta + i \sin 2\theta
 \end{aligned}$$

$$\begin{aligned}
 161. \quad z &= r(\cos \theta + i \sin \theta) \\
 -z &= -r(\cos \theta + i \sin \theta) \\
 &= r(-\cos \theta - i \sin \theta) \\
 &= r(\cos(\theta + \pi) + i \sin(\theta + \pi))
 \end{aligned}$$

 162. Let $a = 0$ and $b = \pi$ in Euler's Formula:

$$\begin{aligned}
 e^{a+bi} &= e^a(\cos b + i \sin b) \\
 e^{0+\pi i} &= e^0(\cos \pi + i \sin \pi) \\
 e^{\pi i} &= -1 \\
 e^{\pi i} + 1 &= 0
 \end{aligned}$$

$$\begin{aligned}
 163. \quad d &= 16 \cos\left(\frac{\pi}{4}t\right) \\
 \text{Maximum displacement: } &16 \\
 \text{Lowest possible } t\text{-value: } &\frac{\pi}{4}t = \frac{\pi}{2} \Rightarrow t = 2
 \end{aligned}$$

$$\begin{aligned}
 164. \quad \text{Maximum displacement: } &\frac{1}{16} \\
 \text{Lowest positive value: } &t = \frac{4}{5}
 \end{aligned}$$

$$\begin{aligned}
 165. \quad d &= \frac{1}{8} \cos(12\pi t) \\
 \text{Maximum displacement: } &\frac{1}{8} \\
 \text{Lowest possible } t\text{-value: } &12\pi t = \frac{\pi}{2} \Rightarrow t = \frac{1}{24}
 \end{aligned}$$

$$\begin{aligned}
 166. \quad \text{Maximum displacement: } &\frac{1}{12} \\
 \text{Lowest positive value: } &t = \frac{1}{60}
 \end{aligned}$$

$$167. 2 \cos(x + \pi) + 2 \cos(x - \pi) = 0$$

$$4 \cos x \cos \pi = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$168. \sin\left(x + \frac{3\pi}{2}\right) - \sin\left(x - \frac{3\pi}{2}\right) = 0$$

$$-\cos x - \cos x = 0$$

$$2 \cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$169. \sin\left(x - \frac{\pi}{3}\right) - \sin\left(x + \frac{\pi}{3}\right) = \frac{3}{2}$$

$$\frac{1}{2} \left[\sin\left(x + \frac{\pi}{3}\right) - \sin\left(x - \frac{\pi}{3}\right) \right] = \left(-\frac{3}{2}\right) \frac{1}{2}$$

$$\cos x \sin \frac{\pi}{3} = -\frac{3}{4}$$

$$\cos x \left(\frac{\sqrt{3}}{2}\right) = -\frac{3}{4}$$

$$\cos x = -\frac{\sqrt{3}}{2}$$

$$x = \frac{5\pi}{6}, \frac{7\pi}{6}$$

$$170. \tan(x + \pi) - \cos\left(x + \frac{5\pi}{2}\right) = 0$$

$$\tan x + \sin x = 0$$

$$\sin x \left(\frac{1}{\cos x} + 1\right) = 0$$

$$x = 0, \pi$$

Review Exercises for Chapter 6

$$1. \text{ Given: } A = 32^\circ, B = 50^\circ, a = 16$$

$$C = 180^\circ - 32^\circ - 50^\circ = 98^\circ$$

$$b = \frac{a \sin B}{\sin A} = \frac{16 \sin 50^\circ}{\sin 32^\circ} \approx 23.13$$

$$c = \frac{a \sin C}{\sin A} = \frac{16 \sin 98^\circ}{\sin 32^\circ} \approx 29.90$$

$$2. \text{ Given: } A = 38^\circ, B = 58^\circ, a = 12$$

$$C = 180^\circ - 38^\circ - 58^\circ = 84^\circ$$

$$b = \frac{a \sin B}{\sin A} = \frac{12 \sin 58^\circ}{\sin 38^\circ} \approx 16.53$$

$$c = \frac{a \sin C}{\sin A} = \frac{12 \sin 84^\circ}{\sin 38^\circ} \approx 19.38$$

$$3. \text{ Given: } B = 25^\circ, C = 105^\circ, c = 25$$

$$A = 180^\circ - 25^\circ - 105^\circ = 50^\circ$$

$$b = \frac{c \sin B}{\sin C} = \frac{25 \sin 25^\circ}{\sin 105^\circ} \approx 10.94$$

$$a = \frac{c \sin A}{\sin C} = \frac{25 \sin 50^\circ}{\sin 105^\circ} \approx 19.83$$

$$4. \text{ Given: } B = 20^\circ, C = 115^\circ, c = 30$$

$$A = 180^\circ - 20^\circ - 115^\circ = 45^\circ$$

$$b = \frac{c \sin B}{\sin C} = \frac{30 \sin 20^\circ}{\sin 115^\circ} \approx 11.32$$

$$a = \frac{c \sin A}{\sin C} = \frac{30 \sin 45^\circ}{\sin 115^\circ} \approx 23.41$$

$$5. \text{ Given: } A = 60^\circ 15' = 60.25^\circ, B = 45^\circ 30' = 45.5^\circ, b = 4.8$$

$$C = 180^\circ - 60.25^\circ - 45.5^\circ = 74.25^\circ = 74^\circ 15'$$

$$a = \frac{b \sin A}{\sin B} = \frac{4.8 \sin 60.25^\circ}{\sin 45.5^\circ} \approx 5.84$$

$$c = \frac{b \sin C}{\sin B} = \frac{4.8 \sin 74.25^\circ}{\sin 45.5^\circ} \approx 6.48$$

6. Given: $A = 82^\circ 45' = 82.75^\circ$, $B = 28^\circ 45' = 28.75^\circ$, $b = 40.2$

$$C = 180^\circ - 82.75^\circ - 28.75^\circ = 68.5^\circ = 68^\circ 30'$$

$$a = \frac{b \sin A}{\sin B} = \frac{40.2 \sin 82.75^\circ}{\sin 28.75^\circ} \approx 82.91$$

$$c = \frac{b \sin C}{\sin B} = \frac{40.2 \sin 68.5^\circ}{\sin 28.75^\circ} \approx 77.76$$

7. Given: $A = 75^\circ$, $a = 2.5$, $b = 16.5$

$$\sin B = \frac{b \sin A}{a} = \frac{16.5 \sin 75^\circ}{2.5} \approx 6.375 \Rightarrow \text{no triangle formed}$$

No solution

8. Given: $A = 15^\circ$, $a = 5$, $b = 10$

$$\sin B = \frac{b \sin A}{a} = \frac{10 \sin 15^\circ}{5} \approx 0.5176 \Rightarrow B \approx 31.2^\circ \text{ or } 148.8^\circ$$

$$\text{Case 1: } B \approx 31.2^\circ$$

$$\text{Case 2: } B \approx 148.8^\circ$$

$$C \approx 180^\circ - 15^\circ - 31.2^\circ = 133.8^\circ$$

$$C \approx 180^\circ - 15^\circ - 148.8^\circ = 16.2^\circ$$

$$c = \frac{a \sin C}{\sin A} \approx \frac{5 \sin 133.8^\circ}{\sin 15^\circ} \approx 13.94$$

$$c = \frac{a \sin C}{\sin A} \approx \frac{5 \sin 16.2^\circ}{\sin 15^\circ} \approx 5.4$$

9. Given: $B = 115^\circ$, $a = 9$, $b = 14.5$

$$\sin A = \frac{a \sin B}{b} = \frac{9 \sin 115^\circ}{14.5} \approx 0.5625 \Rightarrow A \approx 34.2^\circ$$

$$C \approx 180^\circ - 115^\circ - 34.2^\circ = 30.8^\circ$$

$$c = \frac{b \sin C}{\sin B} \approx \frac{14.5 \sin 30.8^\circ}{\sin 115^\circ} \approx 8.18$$

10. Given: $B = 150^\circ$, $a = 64$, $b = 10$

$$\sin A = \frac{a \sin B}{b} = \frac{64 \sin 150^\circ}{10} \approx 3.2 \Rightarrow \text{no triangle formed}$$

No solution

11. Given: $C = 50^\circ$, $a = 25$, $c = 22$

$$\sin A = \frac{a \sin C}{c} = \frac{25 \sin 50^\circ}{22} \approx \frac{25(0.7660)}{22} \approx 0.8705 \Rightarrow A \approx 60.5^\circ \text{ or } 119.5^\circ$$

$$\text{Case 1:}$$

$$\text{Case 2:}$$

$$A \approx 60.5^\circ$$

$$A \approx 119.5^\circ$$

$$B \approx 180^\circ - 50^\circ - 60.5^\circ = 69.5^\circ$$

$$B \approx 180^\circ - 50^\circ - 119.5^\circ = 10.5^\circ$$

$$b = \frac{c \sin B}{\sin C} \approx \frac{22(0.9367)}{0.7660} \approx 26.90$$

$$b = \frac{c \sin B}{\sin C} \approx 5.24$$

12. Given:
- $B = 25^\circ$
- ,
- $a = 6.2$
- ,
- $b = 4$

$$\sin A = \frac{a \sin B}{b} \approx \frac{6.2 \sin 25^\circ}{4} \approx 0.6551 \Rightarrow A \approx 40.9^\circ \text{ or } 139.1^\circ$$

Case 1: $A \approx 40.9^\circ$

$C \approx 180^\circ - 25^\circ - 40.9^\circ = 114.1^\circ$

$$c = \frac{b \sin C}{\sin B} \approx \frac{4 \sin 114.1^\circ}{\sin 25^\circ} \approx 8.64$$

Case 2: $A \approx 139.1^\circ$

$C \approx 180^\circ - 25^\circ - 139.1^\circ = 15.9^\circ$

$$c = \frac{b \sin C}{\sin B} \approx \frac{4 \sin 15.9^\circ}{\sin 25^\circ} \approx 2.60$$

- 13.
- $A = 27^\circ$
- ,
- $b = 5$
- ,
- $c = 8$

Area = $\frac{1}{2}bc \sin A$

$= \frac{1}{2}(5)(8)(\sin 27^\circ)$

≈ 9.08 square units

- 14.
- $B = 80^\circ$
- ,
- $a = 4$
- ,
- $c = 8$

Area = $\frac{1}{2}ac \sin B$

$= \frac{1}{2}(4)(8)(0.9848)$

$= 15.76$ square units

- 15.
- $C = 122^\circ$
- ,
- $b = 18$
- ,
- $a = 29$

Area = $\frac{1}{2}ab \sin C$

$= \frac{1}{2}(29)(18) \sin 122^\circ$

≈ 221.34 square units

16. Area =
- $\frac{1}{2}ab \sin C$

$= \frac{1}{2}(120)(74) \sin 100^\circ$

≈ 4372.5 square units

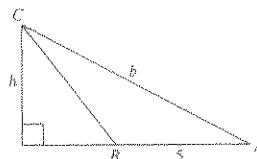
- 17.
- $h = 50 \tan 17^\circ \approx 15.3$
- meters



18. In triangle
- ABC
- ,
- $A = 90^\circ - 62^\circ = 28^\circ$
- ,
- $B = 90^\circ + 38^\circ = 128^\circ$
- , and
- $C = 180^\circ - A - B = 24^\circ$
- .

$$b = \frac{c \sin B}{\sin C} = \frac{5 \sin 128^\circ}{\sin 24^\circ} \approx 9.6870$$

$h = b \sin A = b \sin 28^\circ \approx 4.548 \approx 4.5$ miles



- 19.
- $\sin 28^\circ = \frac{h}{75}$

$h = 75 \sin 28^\circ \approx 35.21$ feet

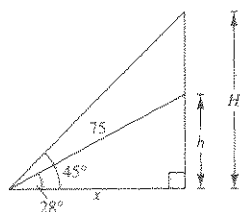
$\cos 28^\circ = \frac{x}{75}$

$x = 75 \cos 28^\circ \approx 66.22$ feet

$\tan 45^\circ = \frac{H}{x}$

$H = x \tan 45^\circ \approx 66.22$ feet

Height of tree: $H - h \approx 31$ feet

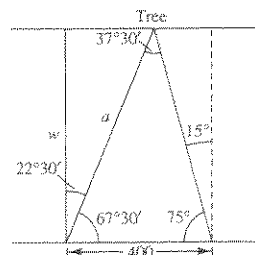


- 20.
- $\frac{a}{\sin 75^\circ} = \frac{400}{\sin 37.5^\circ}$

$$a = \frac{400 \sin 75^\circ}{\sin 37.5^\circ} \approx 634.7$$
 feet

$\sin 67.5^\circ = \frac{w}{a}$

$w = 634.7 \sin 67.5^\circ \approx 586.4$ feet



21. Given: $a = 18, b = 12, c = 15$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{12^2 + 15^2 - 18^2}{2(12)(15)} = 0.125 \Rightarrow A \approx 82.82^\circ$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{18^2 + 15^2 - 12^2}{2(18)(15)} = 0.75 \Rightarrow B \approx 41.41^\circ$$

$$C = 180^\circ - A - B \approx 55.77^\circ$$

22. Given: $a = 10, b = 12, c = 16$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{10^2 + 12^2 - 16^2}{2(10)(12)} = -0.05 \Rightarrow C \approx 92.87^\circ$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{10^2 + 16^2 - 12^2}{2(10)(16)} = 0.6625 \Rightarrow B \approx 48.51^\circ$$

$$A = 180^\circ - B - C \approx 38.62^\circ$$

23. Given: $a = 9, b = 12, c = 20$

$$\begin{aligned} \cos C &= \frac{a^2 + b^2 - c^2}{2ab} \\ &= \frac{81 + 144 - 400}{2(9)(12)} \\ &\approx -0.8102 \Rightarrow C \approx 144.1^\circ \end{aligned}$$

$$\begin{aligned} \sin A &= \frac{a \sin C}{c} \\ &= \frac{9 \sin(144.1^\circ)}{20} \\ &\approx 0.264 \Rightarrow A \approx 15.3^\circ \end{aligned}$$

$$B = 180^\circ - 144.1^\circ - 15.3^\circ = 20.6^\circ$$

24. $a = 7, b = 15, c = 19$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \approx -0.4143 \Rightarrow C \approx 114.5^\circ$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx 0.6955 \Rightarrow B \approx 45.9^\circ$$

$$A = 180^\circ - B - C = 19.6^\circ$$

25. Given: $a = 6.5, b = 10.2, c = 16$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{10.2^2 + 16^2 - 6.5^2}{2(10.2)(16)} \approx 0.97 \Rightarrow A \approx 13.19^\circ$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{6.5^2 + 16^2 - 10.2^2}{2(6.5)(16)} \approx 0.93 \Rightarrow B \approx 20.98^\circ$$

$$C = 180^\circ - A - B \approx 145.83^\circ$$

26. Given: $a = 6.2, b = 6.4, c = 2.1$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{6.4^2 + 2.1^2 - 6.2^2}{2(6.4)(2.1)} \approx 0.26 \Rightarrow A \approx 75.06^\circ$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{6.2^2 + 2.1^2 - 6.4^2}{2(6.2)(2.1)} \approx 0.07 \Rightarrow B \approx 85.84^\circ$$

$$C = 180^\circ - A - B \approx 19.10^\circ$$

27. Given:
- $C = 65^\circ$
- ,
- $a = 25$
- ,
- $b = 12$

$$c^2 = a^2 + b^2 - 2ab \cos C = 25^2 + 12^2 - 2(25)(12) \cos 65^\circ \approx 515.4290 \Rightarrow c \approx 22.70$$

$$\sin A = \frac{a \sin C}{c} = \frac{25 \sin 65^\circ}{22.70} \approx 0.998 \Rightarrow A \approx 86.38^\circ$$

$$B = 180^\circ - A - C \approx 28.62^\circ$$

28. Given:
- $B = 48^\circ$
- ,
- $a = 18$
- ,
- $c = 12$

$$b^2 = a^2 + c^2 - 2ac \cos B = 18^2 + 12^2 - 2(18)(12) \cos 48^\circ \approx 178.9356 \Rightarrow b \approx 13.38$$

$$\sin C = \frac{c \sin B}{b} = \frac{12 \sin 48^\circ}{13.38} \approx 0.666663 \Rightarrow C \approx 41.81^\circ$$

$$A = 180^\circ - B - C \approx 90.19^\circ$$

(Answers may vary.)

29. Given:
- $B = 110^\circ$
- ,
- $a = 4$
- ,
- $c = 4$

$$b^2 = a^2 + c^2 - 2ac \cos B = 16 + 16 - 2(4)(4)(\cos 110^\circ) \approx 42.94 \Rightarrow b \approx 6.55$$

$$\sin A = \frac{a \sin B}{b} \approx \frac{4 \sin 110^\circ}{6.55} \approx 0.5739 \Rightarrow A \approx 35^\circ$$

$$c = a \Rightarrow C = A \approx 35^\circ$$

30. Given:
- $B = 150^\circ$
- ,
- $a = 10$
- ,
- $c = 20$

$$b^2 = a^2 + c^2 - 2ac \cos B \approx 100 + 400 - 400(-0.8660) \approx 846.4 \Rightarrow b \approx 29.09$$

$$\sin C = \frac{c \sin B}{b} \approx \frac{20(0.5)}{29.09} \approx 0.3437 \Rightarrow C \approx 20.1^\circ$$

$$\sin A = \frac{a \sin B}{b} \approx \frac{10(0.5)}{29.09} \approx 0.1719 \Rightarrow A \approx 9.9^\circ$$

31. Given:
- $B = 55^\circ 30' = 55.5^\circ$
- ,
- $a = 12.4$
- ,
- $c = 18.5$

$$b^2 = a^2 + c^2 - 2ac \cos B = 12.4^2 + 18.5^2 - 2(12.4)(18.5) \cos 55.5^\circ \approx 236.1428 \Rightarrow b \approx 15.37$$

$$\sin A = \frac{a \sin B}{b} \approx \frac{12.4 \sin 55.5^\circ}{15.37} \approx 0.665 \Rightarrow A \approx 41.68^\circ \approx 41^\circ 41'$$

$$C = 180^\circ - A - B = 82.82^\circ \approx 82^\circ 49'$$

32. Given:
- $B = 85^\circ 15' = 85.25^\circ$
- ,
- $a = 24.2$
- ,
- $c = 28.2$

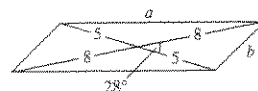
$$b^2 = a^2 + c^2 - 2ac \cos B = 24.2^2 + 28.2^2 - 2(24.2)(28.2) \cos 85.25^\circ \approx 1267.8567 \Rightarrow b \approx 35.61$$

$$\sin A = \frac{a \sin B}{b} \approx \frac{24.2 \sin 85.25^\circ}{35.61} \approx 0.677 \Rightarrow A \approx 42.63^\circ \approx 42^\circ 38'$$

$$C = 180^\circ - A - B = 52.12^\circ \approx 52^\circ 7'$$

- 33.
- $a^2 = 5^2 + 8^2 - 2(5)(8) \cos 152^\circ \approx 159.6 \Rightarrow a \approx 12.63$
- ft

$$b^2 = 5^2 + 8^2 - 2(5)(8) \cos 28^\circ \approx 18.36 \Rightarrow b \approx 4.285$$
 ft



$$34. a^2 = 15^2 + 20^2 - 2(15)(20) \cos(146^\circ) \approx 1122.42 \Rightarrow a \approx 33.5 \text{ m}$$

$$b^2 = 15^2 + 20^2 - 2(15)(20) \cos(34^\circ) \approx 127.58 \Rightarrow b \approx 11.3 \text{ m}$$



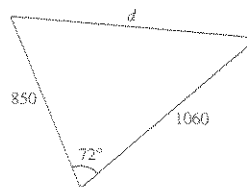
$$35. \text{Angle between planes is } 5^\circ + 67^\circ = 72^\circ.$$

In two hours, distances from airport are 850 miles and 1060 miles.

By the Law of Cosines,

$$d^2 = 850^2 + 1060^2 - 2(850)(1060) \cos(72^\circ) \approx 1,289,251.376$$

$$d \approx 1135.5 \text{ miles.}$$



$$36. b^2 = a^2 + c^2 - 2ac \cos B$$

$$= 300^2 + 425^2 - 2(300)(425) \cos(180^\circ - 65^\circ)$$

$$\approx 378,392.66$$

$$b \approx 615.1 \text{ meters}$$

$$37. a = 4, b = 5, c = 7$$

$$s = \frac{a + b + c}{2} = \frac{4 + 5 + 7}{2} = 8$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{8(4)(3)(1)}$$

$$\approx 9.798 \text{ square units}$$

$$38. a = 15, b = 8, c = 10$$

$$s = \frac{15 + 8 + 10}{2} = 16.5$$

$$\text{Area} = \sqrt{16.5(1.5)(8.5)(6.5)}$$

$$\approx 36.98 \text{ square units}$$

$$39. a = 64.8, b = 49.2, c = 24.1$$

$$s = \frac{a + b + c}{2} = \frac{64.8 + 49.2 + 24.1}{2} = 69.05$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{69.05(4.25)(19.85)(44.95)}$$

$$\approx 511.7 \text{ square units}$$

$$40. s = \frac{8.55 + 5.14 + 12.73}{2} = 13.21$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{13.21(4.66)(8.07)(0.48)}$$

$$\approx 15.4 \text{ square units}$$

$$41. \text{Initial point: } (-5, 4)$$

$$\text{Terminal point: } (2, -1)$$

$$\mathbf{v} = \langle 2 - (-5), -1 - 4 \rangle = \langle 7, -5 \rangle$$

$$42. \text{Initial point: } (0, 1)$$

$$\text{Terminal point: } (6, \frac{7}{2})$$

$$\mathbf{v} = \langle 6 - 0, \frac{7}{2} - 1 \rangle = \langle 6, \frac{5}{2} \rangle$$

$$44. \text{Initial point: } (1, 5)$$

$$\text{Terminal point: } (15, 9)$$

$$\mathbf{v} = \langle 15 - 1, 9 - 5 \rangle = \langle 14, 4 \rangle$$

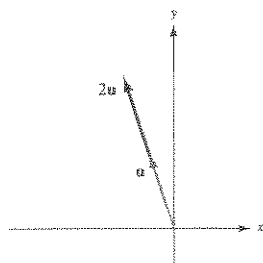
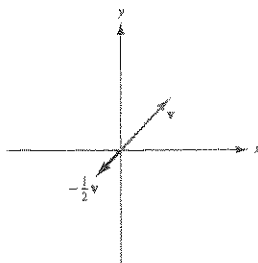
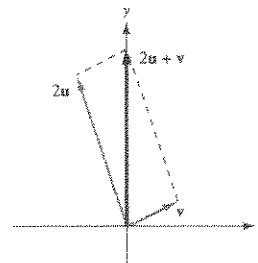
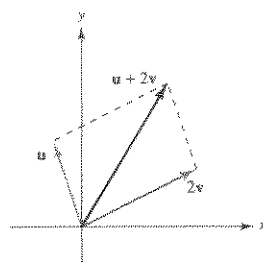
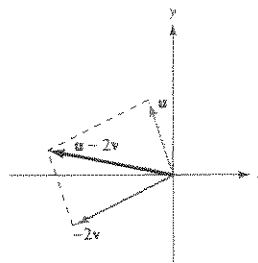
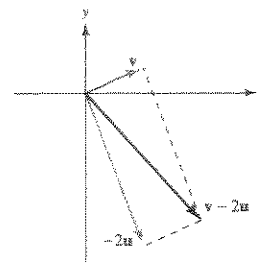
$$43. \text{Initial point: } (0, 10)$$

$$\text{Terminal point: } (7, 3)$$

$$\mathbf{v} = \langle 7 - 0, 3 - 10 \rangle = \langle 7, -7 \rangle$$

$$45. 8 \cos 120^\circ \mathbf{i} + 8 \sin 120^\circ \mathbf{j} = \langle -4, 4\sqrt{3} \rangle$$

$$46. \left\langle \frac{1}{2} \cos 225^\circ, \frac{1}{2} \sin 225^\circ \right\rangle = \left\langle -\frac{\sqrt{2}}{4}, -\frac{\sqrt{2}}{4} \right\rangle$$

47. $2\mathbf{u}$ 48. $-\frac{1}{2}\mathbf{v}$ 49. $2\mathbf{u} + \mathbf{v}$ 50. $\mathbf{u} + 2\mathbf{v}$ 51. $\mathbf{u} - 2\mathbf{v}$ 52. $\mathbf{v} - 2\mathbf{u}$ 

53. (a) $\mathbf{u} + \mathbf{v} = \langle -1, -3 \rangle + \langle -3, 6 \rangle = \langle -4, 3 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle 2, -9 \rangle$

(c) $3\mathbf{u} = \langle -3, -9 \rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = \langle -6, 12 \rangle + \langle -5, -15 \rangle = \langle -11, -3 \rangle$

54. (a) $\mathbf{u} + \mathbf{v} = \langle 4, 5 \rangle + \langle 0, -1 \rangle = \langle 4, 4 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle 4, 6 \rangle$

(c) $3\mathbf{u} = \langle 12, 15 \rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = \langle 0, -2 \rangle + \langle 20, 25 \rangle = \langle 20, 23 \rangle$

55. (a) $\mathbf{u} + \mathbf{v} = \langle -5, 2 \rangle + \langle 4, 4 \rangle = \langle -1, 6 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -9, -2 \rangle$

(c) $3\mathbf{u} = \langle -15, 6 \rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = \langle 8, 8 \rangle + \langle -25, 10 \rangle = \langle -17, 18 \rangle$

56. (a) $\mathbf{u} + \mathbf{v} = \langle 1, -8 \rangle + \langle 3, -2 \rangle = \langle 4, -10 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -2, -6 \rangle$

(c) $3\mathbf{u} = \langle 3, -24 \rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = \langle 6, -4 \rangle + \langle 5, -40 \rangle = \langle 11, -44 \rangle$

57. (a) $\mathbf{u} + \mathbf{v} = \langle 2, -1 \rangle + \langle 5, 3 \rangle = \langle 7, 2 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -3, -4 \rangle$

(c) $3\mathbf{u} = \langle 6, -3 \rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = \langle 10, 6 \rangle + \langle 10, -5 \rangle = \langle 20, 1 \rangle$

58. (a) $\mathbf{u} + \mathbf{v} = \langle 0, -6 \rangle + \langle 1, 1 \rangle = \langle 1, -5 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -1, -7 \rangle$

(c) $3\mathbf{u} = \langle 0, -18 \rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = \langle 2, 2 \rangle + \langle 0, -30 \rangle = \langle 2, -28 \rangle$

59. (a) $\mathbf{u} + \mathbf{v} = \langle 4, 0 \rangle + \langle -1, 6 \rangle = \langle 3, 6 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle 5, -6 \rangle$

(c) $3\mathbf{u} = \langle 12, 0 \rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = \langle -2, 12 \rangle + \langle 20, 0 \rangle = \langle 18, 12 \rangle$

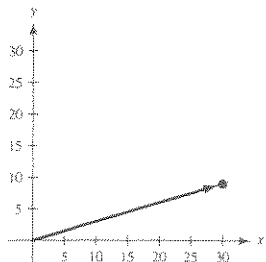
60. (a) $\mathbf{u} + \mathbf{v} = \langle -7, -3 \rangle + \langle 4, -1 \rangle = \langle -3, -4 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -11, -2 \rangle$

(c) $3\mathbf{u} = \langle -21, -9 \rangle$

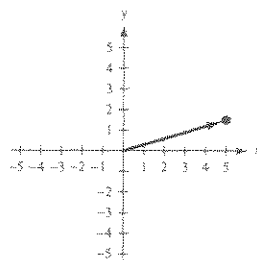
(d) $2\mathbf{v} + 5\mathbf{u} = \langle 8, -2 \rangle + \langle -35, -15 \rangle = \langle -27, -17 \rangle$

61. $3\mathbf{v} = 3(10\mathbf{i} + 3\mathbf{j}) = 30\mathbf{i} + 9\mathbf{j} = \langle 30, 9 \rangle$

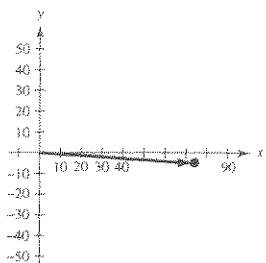


62. $\mathbf{v} = 10\mathbf{i} + 3\mathbf{j}$

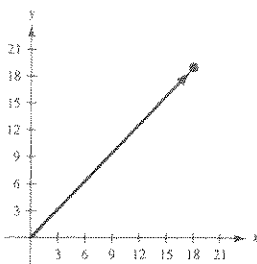
$$\frac{1}{2}\mathbf{v} = 5\mathbf{i} + \frac{3}{2}\mathbf{j}$$



63. $\mathbf{w} = 4\mathbf{u} + 5\mathbf{v} = 4(6\mathbf{i} - 5\mathbf{j}) + 5(10\mathbf{i} + 3\mathbf{j})$
 $= 74\mathbf{i} - 5\mathbf{j}$



64. $3\mathbf{v} - 2\mathbf{u} = 3(10\mathbf{i} + 3\mathbf{j}) - 2(6\mathbf{i} - 5\mathbf{j}) = 18\mathbf{i} + 19\mathbf{j}$



65. $\|\mathbf{u}\| = 6$

Unit vector: $\frac{1}{6}\langle 0, -6 \rangle = \langle 0, -1 \rangle$

66. $\|\mathbf{v}\| = \sqrt{(-12)^2 + (-5)^2} = \sqrt{169} = 13$

Unit vector: $\langle -\frac{12}{13}, -\frac{5}{13} \rangle$

67. $\|\mathbf{v}\| = \sqrt{5^2 + (-2)^2} = \sqrt{29}$

Unit vector: $\frac{1}{\sqrt{29}}\langle 5, -2 \rangle = \langle \frac{5}{\sqrt{29}}, \frac{-2}{\sqrt{29}} \rangle$

68. $\|\mathbf{w}\| = 7$

Unit vector: $\frac{1}{7}\langle -7\mathbf{i} \rangle = -\mathbf{i}$

69. $\mathbf{u} = \langle 1 - (-8), -5 - 3 \rangle = \langle 9, -8 \rangle = 9\mathbf{i} - 8\mathbf{j}$

70. $\mathbf{u} = \langle -6.4 - 2, 10.8 - (-3.2) \rangle = -8.4\mathbf{i} + 14\mathbf{j}$

71. $\mathbf{v} = -10\mathbf{i} + 10\mathbf{j}$

$$\|\mathbf{v}\| = \sqrt{(-10)^2 + (10)^2} = \sqrt{200} = 10\sqrt{2}$$

$$\tan \theta = \frac{10}{-10} = -1 \implies \theta = 135^\circ \text{ since } \mathbf{v} \text{ is in Quadrant II.}$$

$$\mathbf{v} = 10\sqrt{2}(\cos 135^\circ \mathbf{i} + \sin 135^\circ \mathbf{j})$$

72. $\mathbf{v} = 4\mathbf{i} - \mathbf{j}$

$$\|\mathbf{v}\| = \sqrt{4^2 + (-1)^2} = \sqrt{17}$$

$$\tan \theta = \frac{-1}{4} = -\frac{1}{4} \implies \theta \approx 346^\circ, \text{ since } \theta \text{ is in Quadrant IV.}$$

$$\mathbf{v} = \sqrt{17}(\cos 346^\circ \mathbf{i} + \sin 346^\circ \mathbf{j})$$

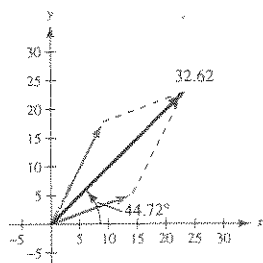
73. $\mathbf{u} = 15[(\cos 20^\circ)\mathbf{i} + (\sin 20^\circ)\mathbf{j}]$

$\mathbf{v} = 20[(\cos 63^\circ)\mathbf{i} + (\sin 63^\circ)\mathbf{j}]$

$\mathbf{u} + \mathbf{v} \approx 23.1752\mathbf{i} + 22.9504\mathbf{j}$

$\|\mathbf{u} + \mathbf{v}\| \approx 32.62$

$\tan \theta = \frac{22.9504}{23.1752} \Rightarrow \theta \approx 44.72^\circ$



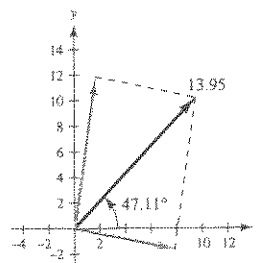
74. $\mathbf{u} = 12[(\cos 82^\circ)\mathbf{i} + (\sin 82^\circ)\mathbf{j}]$

$\mathbf{v} = 8[(\cos(-12^\circ))\mathbf{i} + (\sin(-12^\circ))\mathbf{j}]$

$\mathbf{u} + \mathbf{v} \approx 9.4953\mathbf{i} + 10.2199\mathbf{j}$

$\|\mathbf{u} + \mathbf{v}\| \approx 13.95$

$\tan \theta = \frac{10.2199}{9.4953} \Rightarrow \theta \approx 47.11^\circ$



75. $\mathbf{F}_1 = 250\langle \cos 60^\circ, \sin 60^\circ \rangle$

$\mathbf{F}_2 = 100\langle \cos 150^\circ, \sin 150^\circ \rangle$

$\mathbf{F}_3 = 200\langle \cos(-90^\circ), \sin(-90^\circ) \rangle$

$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \approx \langle 38.39746, 66.50635 \rangle$

$\tan \theta \approx \frac{66.50635}{38.39746} \Rightarrow \theta \approx 60^\circ$

$\|\mathbf{F}\| = \sqrt{38.39746^2 + 66.50635^2} \approx 76.8 \text{ pounds}$

76. Force One: $\mathbf{u} = 85\mathbf{i}$

Force Two: $\mathbf{v} = 50 \cos 15^\circ \mathbf{i} + 50 \sin 15^\circ \mathbf{j}$

Resultant Force:

$\mathbf{u} + \mathbf{v} = (85 + 50 \cos 15^\circ)\mathbf{i} + (50 \sin 15^\circ)\mathbf{j}$

$$\begin{aligned} \|\mathbf{u} + \mathbf{v}\| &= \sqrt{(85 + 50 \cos 15^\circ)^2 + (50 \sin 15^\circ)^2} \\ &= \sqrt{85^2 + 8500 \cos 15^\circ + 50^2} \\ &= 133.92 \text{ lb} \end{aligned}$$

$\tan \theta = \frac{50 \sin 15^\circ}{85 + 50 \cos 15^\circ} \Rightarrow \theta \approx 5.5^\circ \text{ from the 85-pound force.}$

77. Rope One: $\mathbf{u} = \|\mathbf{u}\|(\cos 30^\circ \mathbf{i} - \sin 30^\circ \mathbf{j}) = \|\mathbf{u}\|\left(\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right)$

Rope Two: $\mathbf{v} = \|\mathbf{u}\|(-\cos 30^\circ \mathbf{i} - \sin 30^\circ \mathbf{j}) = \|\mathbf{u}\|\left(-\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right)$

Resultant: $\mathbf{u} + \mathbf{v} = -\|\mathbf{u}\|\mathbf{j} = -180\mathbf{j}$

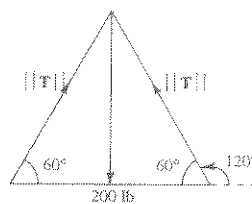
$\|\mathbf{u}\| = 180$

Therefore, the tension on each rope is $\|\mathbf{u}\| = 180$ pounds.

78. By symmetry, the magnitudes of the tensions are equal.

$$\mathbf{T} = \|\mathbf{T}\|(\cos 120^\circ \mathbf{i} + \sin 120^\circ \mathbf{j})$$

$$\|\mathbf{T}\| \sin 120^\circ = \frac{1}{2}(200) \Rightarrow \|\mathbf{T}\| = \frac{100}{\sqrt{3}/2} = \frac{200}{\sqrt{3}} \approx 115.5 \text{ lbs}$$



79. Airplane velocity: $\mathbf{u} = 430\langle \cos 315^\circ, \sin 315^\circ \rangle$

Wind velocity: $\mathbf{w} = 35\langle \cos 60^\circ, \sin 60^\circ \rangle$

$$\mathbf{u} + \mathbf{w} \approx \langle 321.5559, -273.7450 \rangle$$

$$\|\mathbf{u} + \mathbf{w}\| \approx 422.3 \text{ mph}$$

$$\tan(\mathbf{u} + \mathbf{w}) = \frac{-273.7450}{321.5559}$$

$$\approx -0.8513 \Rightarrow \theta \approx -40.4^\circ$$

The bearing from the north is $90^\circ + 40.4^\circ = 130.4^\circ$.

80. Airspeed: $\mathbf{u} = 724(\cos 60^\circ \mathbf{i} + \sin 60^\circ \mathbf{j})$

$$= 362(\mathbf{i} + \sqrt{3}\mathbf{j})$$

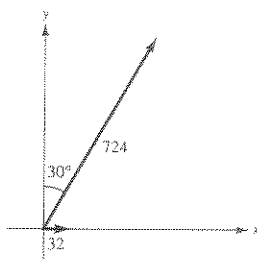
Wind: $\mathbf{w} = 32\mathbf{i}$

$$\text{Groundspeed} = \mathbf{u} + \mathbf{w} = (394\mathbf{i} + 362\sqrt{3}\mathbf{j})$$

$$\|\mathbf{u} + \mathbf{w}\| = \sqrt{(394)^2 + (362\sqrt{3})^2} \approx 740.5 \text{ km/hr}$$

$$\tan \theta = \frac{362\sqrt{3}}{394} \Rightarrow \theta \approx 57.9^\circ$$

Bearing: N 32.1° E (or 32.1° in airplane navigation)



$$81. \mathbf{u} \cdot \mathbf{v} = \langle 0, -2 \rangle \cdot \langle 1, 10 \rangle = 0 - 20 = -20$$

$$83. \mathbf{u} \cdot \mathbf{v} = \langle 6, -1 \rangle \cdot \langle 2, 5 \rangle = 6(2) + (-1)(5) = 7$$

$$85. \mathbf{u} \cdot \mathbf{u} = \langle -3, -4 \rangle \cdot \langle -3, -4 \rangle \\ = 9 + 16 = 25 = \|\mathbf{u}\|^2$$

$$87. 4\mathbf{u} \cdot \mathbf{v} = 4\langle -3, -4 \rangle \cdot \langle 2, 1 \rangle = 4(-6 - 4) = -40$$

$$89. \mathbf{u} = \langle 2\sqrt{2}, -4 \rangle, \mathbf{v} = \langle -\sqrt{2}, 1 \rangle$$

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{-8}{(\sqrt{24})(\sqrt{3})} \Rightarrow \theta \approx 160.5^\circ$$

$$91. \mathbf{u} = \cos \frac{7\pi}{4} \mathbf{i} + \sin \frac{7\pi}{4} \mathbf{j} = \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$$

$$\mathbf{v} = \cos \frac{5\pi}{6} \mathbf{i} + \sin \frac{5\pi}{6} \mathbf{j} = \left\langle -\frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$$

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{(-\sqrt{3}/(2\sqrt{2})) - (1/(2\sqrt{2}))}{(1)(1)}$$

$$\approx -0.966 \Rightarrow \theta \approx 165^\circ \text{ or } \frac{11\pi}{12}$$

$$82. \mathbf{u} \cdot \mathbf{v} = \langle -4, 5 \rangle \cdot \langle 3, -1 \rangle = -12 - 5 = -17$$

$$84. \mathbf{u} \cdot \mathbf{v} = (8\mathbf{i} - 7\mathbf{j}) \cdot (3\mathbf{i} - 4\mathbf{j}) = 24 + 28 = 52$$

$$86. \|\mathbf{v}\| - 3 = \sqrt{(2)^2 + (1)^2} - 3 = \sqrt{5} - 3$$

$$88. (\mathbf{u} \cdot \mathbf{v})\mathbf{u} = (-10)\langle -3, -4 \rangle = \langle 30, 40 \rangle$$

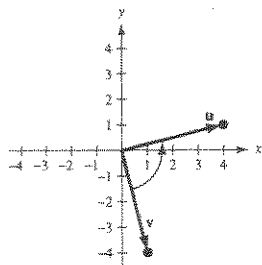
$$90. \mathbf{u} = \langle 3, 1 \rangle, \mathbf{v} = \langle 4, 5 \rangle$$

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{17}{(\sqrt{10})(\sqrt{41})} \Rightarrow \theta \approx 32.9^\circ$$

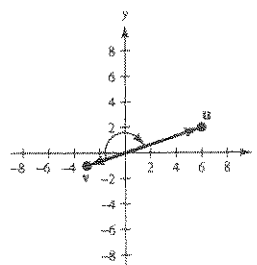
$$92. \text{Angle} = 45^\circ + 60^\circ = 105^\circ$$

$$93. \cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$$

$$= 0 \Rightarrow \theta = 90^\circ$$



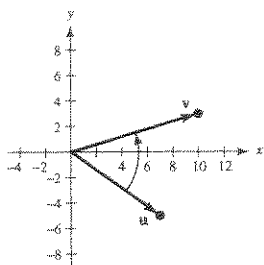
$$94. 180^\circ$$



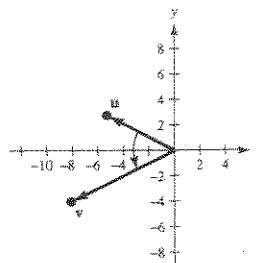
$$95. \cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{70 - 15}{\sqrt{74} \sqrt{109}}$$

$$\approx 0.612$$

$$\Rightarrow \theta \approx 52.2^\circ$$



$$96. 54.1^\circ$$



$$97. \mathbf{u} = \langle 39, -12 \rangle, \mathbf{v} = \langle -26, 8 \rangle$$

$$\mathbf{u} \cdot \mathbf{v} = 39(-26) + (-12)(8)$$

$$= -1110 \neq 0 \Rightarrow \mathbf{u} \text{ and } \mathbf{v} \text{ are not orthogonal.}$$

$$\mathbf{v} = -\frac{2}{3}\mathbf{u} \Rightarrow \mathbf{u} \text{ and } \mathbf{v} \text{ are parallel.}$$

$$98. \mathbf{u} \cdot \mathbf{v} = \langle 8, -4 \rangle \cdot \langle 5, 10 \rangle = 40 - 40 = 0 \Rightarrow \text{orthogonal}$$

$$99. \mathbf{u} = \langle 8, 5 \rangle, \mathbf{v} = \langle -2, 4 \rangle$$

$$\mathbf{u} \cdot \mathbf{v} = 8(-2) + (5)(4)$$

$$= 4 \neq 0 \Rightarrow \mathbf{u} \text{ and } \mathbf{v} \text{ are not orthogonal.}$$

$$\mathbf{u} \neq k\mathbf{v} \Rightarrow \mathbf{u} \text{ and } \mathbf{v} \text{ are not parallel.}$$

Neither

$$100. -\frac{3}{4}\mathbf{v} = -\frac{3}{4}\langle 20, -68 \rangle = \langle -15, 51 \rangle = \mathbf{u} \Rightarrow \text{parallel}$$

$$101. \langle 1, -k \rangle \cdot \langle 1, 2 \rangle = 1 - 2k = 0 \Rightarrow k = \frac{1}{2}$$

$$102. \langle 2, 1 \rangle \cdot \langle -1, -k \rangle = -2 - k = 0 \Rightarrow k = -2$$

$$103. \langle k, -1 \rangle \cdot \langle 2, -2 \rangle = 2k + 2 = 0 \Rightarrow k = -1$$

$$104. \langle k, -2 \rangle \cdot \langle 1, 4 \rangle = k - 8 = 0 \Rightarrow k = 8$$

$$105. \mathbf{u} = \langle -4, 3 \rangle, \mathbf{v} = \langle -8, -2 \rangle$$

$$\text{proj}_{\mathbf{v}} \mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v} = \left(\frac{26}{68} \right) \langle -8, -2 \rangle = -\frac{13}{17} \langle 4, 1 \rangle$$

$$\begin{aligned} \mathbf{u} - \text{proj}_{\mathbf{v}} \mathbf{u} &= \langle -4, 3 \rangle - \left\langle \frac{-52}{17}, \frac{-13}{17} \right\rangle \\ &= \left\langle -\frac{16}{17}, \frac{64}{17} \right\rangle \end{aligned}$$

$$\mathbf{u} = \left\langle -\frac{52}{17}, -\frac{13}{17} \right\rangle + \left\langle -\frac{16}{17}, \frac{64}{17} \right\rangle$$

$$107. \mathbf{u} = \langle 2, 7 \rangle, \mathbf{v} = \langle 1, -1 \rangle$$

$$\text{proj}_{\mathbf{v}} \mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v} = \frac{-5}{2} \langle 1, -1 \rangle = \left\langle -\frac{5}{2}, \frac{5}{2} \right\rangle$$

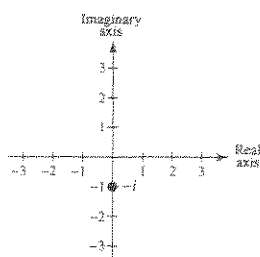
$$\mathbf{u} - \text{proj}_{\mathbf{v}} \mathbf{u} = \langle 2, 7 \rangle - \left\langle -\frac{5}{2}, \frac{5}{2} \right\rangle = \left\langle \frac{9}{2}, \frac{9}{2} \right\rangle$$

$$\mathbf{u} = \left\langle -\frac{5}{2}, \frac{5}{2} \right\rangle + \left\langle \frac{9}{2}, \frac{9}{2} \right\rangle$$

$$109. 48 \text{ inches} = 4 \text{ feet}$$

$$\text{Work} = 18,000(4) = 72,000 \text{ ft} \cdot \text{lb}$$

$$111. |-i| = 1$$



$$106. \mathbf{u} = \langle 5, 6 \rangle, \mathbf{v} = \langle 10, 0 \rangle$$

$$\text{proj}_{\mathbf{v}} \mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v} = \frac{50}{100} \langle 10, 0 \rangle = \langle 5, 0 \rangle$$

$$\mathbf{u} = \langle 5, 0 \rangle + \langle 0, 6 \rangle$$

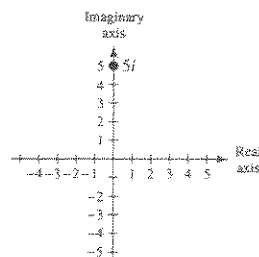
$$108. \mathbf{u} = \langle -3, 5 \rangle, \mathbf{v} = \langle -5, 2 \rangle$$

$$\text{proj}_{\mathbf{v}} \mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v} = \frac{25}{29} \langle -5, 2 \rangle$$

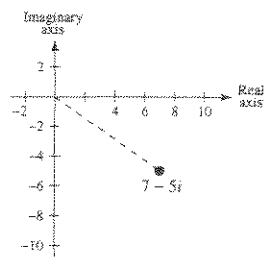
$$\mathbf{u} = \left\langle -\frac{125}{29}, \frac{50}{29} \right\rangle + \left\langle \frac{38}{29}, \frac{95}{29} \right\rangle$$

$$110. \text{Force} = 500 \sin 12^\circ \approx 104 \text{ lbs}$$

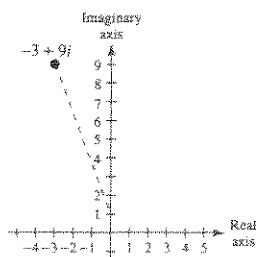
$$112. |5i| = 5$$



$$113. |7 - 5i| = \sqrt{7^2 + (-5)^2} = \sqrt{74}$$



$$114. |-3 + 9i| = \sqrt{9 + 81} = \sqrt{90} = 3\sqrt{10}$$



$$115. z = 2 - 2i$$

$$|z| = \sqrt{4 + 4} = 2\sqrt{2}$$

$$\theta = \frac{7\pi}{4}$$

$$z = 2\sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$$

$$116. z = -2 + 2i$$

$$|z| = \sqrt{4 + 4} = 2\sqrt{2}$$

$$\theta = \frac{3\pi}{4}$$

$$z = 2\sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$

117. $z = -\sqrt{3} - i$

$|z| = \sqrt{3 + 1} = 2$

$\theta = \frac{7\pi}{6}$

$z = 2\left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}\right)$

118. $z = -\sqrt{3} + i$

$|z| = \sqrt{3 + 1} = 2$

$\theta = \frac{5\pi}{6}$

$z = 2\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right)$

119. $z = -2i$

$|z| = 2$

$\theta = \frac{3\pi}{2}$

$z = 2\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$

120. $z = 4i$

$|z| = 4$

$\theta = \frac{\pi}{2}$

$z = 4\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$

121. $\left[\frac{5}{2}\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)\right]\left[4\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)\right] = 10\left[\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right]$

122. $6\left[\cos\left(\frac{2\pi}{3} + \frac{\pi}{6}\right) + i \sin\left(\frac{2\pi}{3} + \frac{\pi}{6}\right)\right] = 6\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right) = -3\sqrt{3} + 3i$

123. $\frac{20(\cos 320^\circ + i \sin 320^\circ)}{5(\cos 80^\circ + i \sin 80^\circ)} = 4[\cos 240^\circ + i \sin 240^\circ]$

124. $\frac{3}{9}[\cos(230^\circ - 95^\circ) + i \sin(230^\circ - 95^\circ)] = \frac{1}{3}(\cos 135^\circ + i \sin 135^\circ)$
 $= -\frac{\sqrt{2}}{6} + \frac{\sqrt{2}}{6}i$

125. (a) $2 - 2i = 2\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$

$3 + 3i = 3\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$

(b) $2\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)3\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) = 12(\cos 2\pi + i \sin 2\pi) = 12$

(c) $(2 - 2i)(3 + 3i) = 6 + 6 = 12$

126. (a) $4 + 4i = 4\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$

$-1 - i = \sqrt{2}\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right)$

(b) $4\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)\sqrt{2}\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right) = 8\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right) = -8i$

(c) $(4 + 4i)(-1 - i) = -4 - 4i - 4i + 4 = -8i$

$$127. (a) -i = \cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}$$

$$2 + 2i = 2\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

$$\begin{aligned} (b) \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right) 2\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) &= 2\sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right) \\ &= 2\sqrt{2} \left(\frac{\sqrt{2}}{2} + i \left(-\frac{\sqrt{2}}{2} \right) \right) \\ &= 2 - 2i \end{aligned}$$

$$(c) -i(2 + 2i) = -2i + 2 = 2 - 2i$$

$$128. (a) 4i = 4 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$

$$1 - i = \sqrt{2} \left(\cos \left(-\frac{\pi}{4} \right) + i \sin \left(-\frac{\pi}{4} \right) \right)$$

$$\begin{aligned} (b) 4 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) \sqrt{2} \left(\cos \left(-\frac{\pi}{4} \right) + i \sin \left(-\frac{\pi}{4} \right) \right) &= 4\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) \\ &= 4\sqrt{2} \left(\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right) \\ &= 4 + 4i \end{aligned}$$

$$(c) 4i(1 - i) = 4i + 4 = 4 + 4i$$

$$129. (a) 3 - 3i = 3\sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$$

$$2 + 2i = 2\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

$$\begin{aligned} (b) \frac{3\sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)}{2\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)} &= \frac{3}{2} \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right) \\ &= \frac{3}{2}(-i) = -\frac{3}{2}i \end{aligned}$$

$$(c) \frac{3 - 3i}{2 + 2i} \cdot \frac{(2 - 2i)}{(2 - 2i)} = \frac{6 - 12i - 6}{8} = -\frac{12i}{8} = -\frac{3}{2}i$$

$$130. (a) -1 - i = \sqrt{2} \left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right)$$

$$-2 - 2i = 2\sqrt{2} \left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right)$$

$$(b) \frac{-1 - i}{-2 - 2i} = \frac{\sqrt{2}}{2\sqrt{2}} (\cos 0 + i \sin 0) = \frac{1}{2}$$

$$(c) \frac{-1 - i}{-2 - 2i} = \frac{-1 - i}{2(-1 - i)} = \frac{1}{2}$$

$$\begin{aligned}
 131. \left[5 \left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right) \right]^4 &= 5^4 \left(\cos \frac{4\pi}{12} + i \sin \frac{4\pi}{12} \right) \\
 &= 625 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \\
 &= 625 \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) \\
 &= \frac{625}{2} + \frac{625\sqrt{3}}{2}i
 \end{aligned}$$

$$\begin{aligned}
 132. \left[2 \left(\cos \frac{4\pi}{15} + i \sin \frac{4\pi}{15} \right) \right]^5 &= 2^5 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right) \\
 &= 32 \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i \right) \\
 &= -16 - 16\sqrt{3}i
 \end{aligned}$$

$$\begin{aligned}
 133. (2 + 3i)^6 &\approx [\sqrt{13}(\cos 56.3^\circ + i \sin 56.3^\circ)]^6 \\
 &= 13^3(\cos 337.9^\circ + i \sin 337.9^\circ) \\
 &\approx 13^3(0.9263 - 0.3769i) \\
 &\approx 2035 - 828i
 \end{aligned}$$

$$\begin{aligned}
 134. (1 - i)^8 &= [\sqrt{2}(\cos 315^\circ + i \sin 315^\circ)]^8 \\
 &= 16(\cos 2520^\circ + i \sin 2520^\circ) \\
 &= 16(\cos 0^\circ + i \sin 0^\circ) \\
 &= 16
 \end{aligned}$$

$$135. -\sqrt{3} + i = 2 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$$

Square roots:

$$\begin{aligned}
 \sqrt{2} \left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right) &\approx 0.3660 + 1.3660i \\
 \sqrt{2} \left(\cos \frac{17\pi}{12} + i \sin \frac{17\pi}{12} \right) &\approx -0.3660 - 1.3660i
 \end{aligned}$$

$$136. \sqrt{3} - i = 2 \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right)$$

Square roots:

$$\begin{aligned}
 \sqrt{2} \left(\cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12} \right) &\approx -1.3660 + 0.3660i \\
 \sqrt{2} \left(\cos \frac{23\pi}{12} + i \sin \frac{23\pi}{12} \right) &\approx 1.3660 - 0.3660i
 \end{aligned}$$

$$137. -2i = 2 \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right)$$

Square roots:

$$\begin{aligned}
 \sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) &= \sqrt{2} \left(-\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right) = -1 + i \\
 \sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right) &= 1 - i
 \end{aligned}$$

$$138. -5i = 5 \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right)$$

Square roots:

$$\begin{aligned}
 \sqrt{5} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) &= \sqrt{5} \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right) = -\frac{\sqrt{10}}{2} + \frac{\sqrt{10}}{2}i \\
 \sqrt{5} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right) &= \frac{\sqrt{10}}{2} - \frac{\sqrt{10}}{2}i
 \end{aligned}$$

$$139. -2 - 2i = 2\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$$

Square roots:

$$2^{3/4}\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right) \approx -0.6436 + 1.5538i$$

$$2^{3/4}\left(\cos\frac{13\pi}{8} + i\sin\frac{13\pi}{8}\right) \approx 0.6436 - 1.5538i$$

$$140. -2 + 2i = 2\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

Square roots:

$$2^{3/4}\left(\cos\frac{3\pi}{8} + i\sin\frac{3\pi}{8}\right) \approx 0.6436 + 1.5538i$$

$$2^{3/4}\left(\cos\frac{11\pi}{8} + i\sin\frac{11\pi}{8}\right) \approx -0.6436 - 1.5538i$$

$$141. (a) \text{ Sixth roots of } -729i = 729\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right):$$

$$\sqrt[6]{729}\left(\cos\frac{(3\pi/2) + 2k\pi}{6} + i\sin\frac{(3\pi/2) + 2k\pi}{6}\right), k = 0, 1, 2, 3, 4, 5$$

$$3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$3\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$$

$$3\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$$

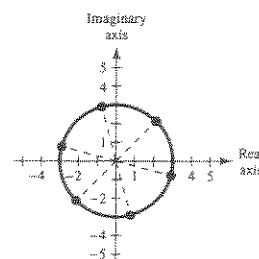
$$3\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$$

$$3\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right)$$

$$3\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right)$$

$$(c) \frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i, -0.7765 + 2.898i, -2.898 + 0.7765i, -\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i, 0.7765 - 2.898i, 2.898 - 0.7765i$$

(b)



$$142. (a) \text{ Fourth roots of } 256i = 256\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right):$$

$$\sqrt[4]{256}\left(\cos\frac{(\pi/2) + 2k\pi}{4} + i\sin\frac{(\pi/2) + 2k\pi}{4}\right), k = 0, 1, 2, 3$$

$$4\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$$

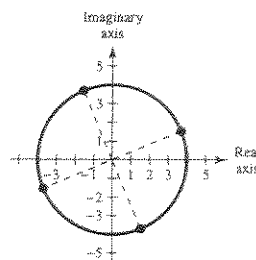
$$4\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right)$$

$$4\left(\cos\frac{9\pi}{8} + i\sin\frac{9\pi}{8}\right)$$

$$4\left(\cos\frac{13\pi}{8} + i\sin\frac{13\pi}{8}\right)$$

$$(c) 3.696 + 1.531i, -1.531 + 3.696i, -3.696 - 1.531i, 1.531 - 3.696i$$

(b)



143. (a) Cube roots of
- $8 = 8(\cos 0 + i \sin 0)$
- :

$$\sqrt[3]{8} \left(\cos \left(\frac{2k\pi}{3} \right) + i \sin \left(\frac{2k\pi}{3} \right) \right), k = 0, 1, 2$$

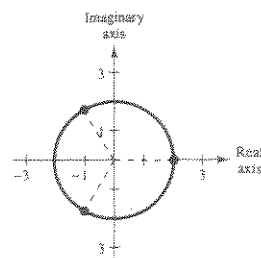
$$2(\cos 0 + i \sin 0)$$

$$2 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

$$2 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right)$$

$$(c) 2, -1 + \sqrt{3}i, -1 - \sqrt{3}i$$

(b)



144. (a) Fifth roots of
- $-1024 = 1024(\cos \pi + i \sin \pi)$
- :

$$\sqrt[5]{1024} \left(\cos \left(\frac{\pi + 2\pi k}{5} \right) + i \sin \left(\frac{\pi + 2\pi k}{5} \right) \right), k = 0, 1, 2, 3, 4$$

$$4 \left(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5} \right)$$

$$4 \left(\cos \frac{3\pi}{5} + i \sin \frac{3\pi}{5} \right)$$

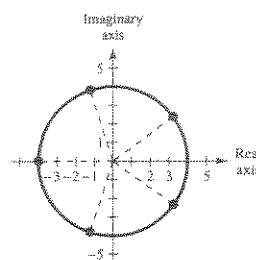
$$4 \left(\cos \frac{5\pi}{5} + i \sin \frac{5\pi}{5} \right) = -4$$

$$4 \left(\cos \frac{7\pi}{5} + i \sin \frac{7\pi}{5} \right)$$

$$4 \left(\cos \frac{9\pi}{5} + i \sin \frac{9\pi}{5} \right)$$

$$(c) 3.236 \pm 2.351i, -1.236 \pm 3.804i, -4$$

(b)



- 145.
- $x^4 + 256 = 0$

$$x^4 = -256 = 256(\cos \pi + i \sin \pi)$$

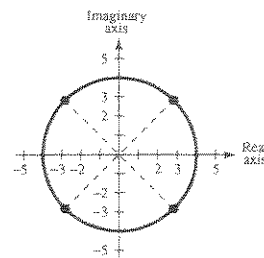
$$\sqrt[4]{-256} = 4 \left[\cos \left(\frac{\pi + 2\pi k}{4} \right) + i \sin \left(\frac{\pi + 2\pi k}{4} \right) \right], k = 0, 1, 2, 3$$

$$4 \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) = \frac{4\sqrt{2}}{2} + \frac{4\sqrt{2}}{2}i = 2\sqrt{2} + 2\sqrt{2}i$$

$$4 \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) = -\frac{4\sqrt{2}}{2} + \frac{4\sqrt{2}}{2}i = -2\sqrt{2} + 2\sqrt{2}i$$

$$4 \left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right) = -\frac{4\sqrt{2}}{2} - \frac{4\sqrt{2}}{2}i = -2\sqrt{2} - 2\sqrt{2}i$$

$$4 \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right) = \frac{4\sqrt{2}}{2} - \frac{4\sqrt{2}}{2}i = 2\sqrt{2} - 2\sqrt{2}i$$



$$146. x^5 = 32i = 32\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$$

$$\sqrt[5]{32}\left(\cos\left(\frac{\pi/2 + 2k\pi}{5}\right) + i \sin\left(\frac{\pi/2 + 2k\pi}{5}\right)\right), k = 0, 1, 2, 3, 4$$

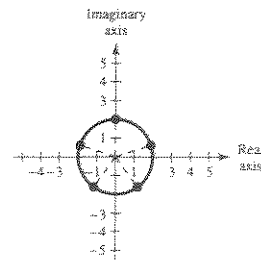
$$2\left(\cos \frac{\pi}{10} + i \sin \frac{\pi}{10}\right)$$

$$2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) = 2i$$

$$2\left(\cos \frac{9\pi}{10} + i \sin \frac{9\pi}{10}\right)$$

$$2\left(\cos \frac{13\pi}{10} + i \sin \frac{13\pi}{10}\right)$$

$$2\left(\cos \frac{17\pi}{10} + i \sin \frac{17\pi}{10}\right)$$



$$147. x^3 + 8i = 0$$

$$x^3 = -8i$$

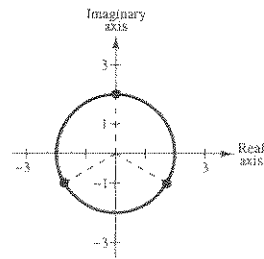
$$-8i = 8\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$$

$$\sqrt[3]{-8i} = \sqrt[3]{8}\left[\cos \frac{(3\pi/2) + 2\pi k}{3} + i \sin \frac{(3\pi/2) + 2\pi k}{3}\right], k = 0, 1, 2$$

$$2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) = 2i$$

$$2\left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}\right) = -\sqrt{3} - i$$

$$2\left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right) = \sqrt{3} - i$$



$$148. x^4 = -81 = 81(\cos \pi + i \sin \pi)$$

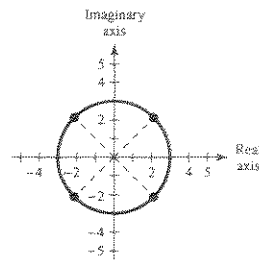
$$\sqrt[4]{81}\left(\cos \frac{\pi + 2k\pi}{4} + i \sin \frac{\pi + 2k\pi}{4}\right), k = 0, 1, 2, 3$$

$$3\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$$

$$3\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$$

$$3\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right)$$

$$3\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$$



149. True

150. False. There may be no solution, one solution, or two solutions.

151. Length and direction characterize vectors in plane.

152. A and C appear equivalent.

$$\begin{aligned}
 153. \quad z_1 z_2 &= 2(\cos \theta + i \sin \theta) 2(\cos(\pi - \theta) + i \sin(\pi - \theta)) \\
 &= 4(\cos \theta + i \sin \theta)(-\cos \theta + i \sin \theta) \\
 &= 4(-\cos^2 \theta - \sin^2 \theta) = -4
 \end{aligned}$$

$$\begin{aligned}
 \frac{z_1}{z_2} &= \frac{2(\cos \theta + i \sin \theta)}{2(\cos(\pi - \theta) + i \sin(\pi - \theta))} \\
 &= \frac{\cos \theta + i \sin \theta}{-\cos \theta + i \sin \theta} \cdot \frac{-\cos \theta - i \sin \theta}{-\cos \theta - i \sin \theta} \\
 &= -\cos^2 \theta + \sin^2 \theta - 2 \sin \theta \cos \theta i \\
 &= -(\cos \theta + i \sin \theta)^2 \\
 &= -\left(\frac{z_1}{2}\right)^2 \\
 &= -\frac{z_1^2}{4}
 \end{aligned}$$

154. (a) Three roots are not shown.

(b) The modulus of each is 2, and the arguments are 120° , 210° and 300° .

Chapter 6 Practice Test

For Exercises 1 and 2, use the Law of Sines to find the remaining sides and angles of the triangle.

1. $A = 40^\circ$, $B = 12^\circ$, $b = 100$
2. $C = 150^\circ$, $a = 5$, $c = 20$
3. Find the area of the triangle: $a = 3$, $b = 6$, $C = 130^\circ$
4. Determine the number of solutions to the triangle: $a = 10$, $b = 35$, $A = 22.5^\circ$

For Exercises 5 and 6, use the Law of Cosines to find the remaining sides and angles of the triangle.

5. $a = 49$, $b = 53$, $c = 38$
6. $C = 29^\circ$, $a = 100$, $b = 300$
7. Use Heron's Formula to find the area of the triangle: $a = 4.1$, $b = 6.8$, $c = 5.5$.
8. A ship travels 40 miles due east, then adjusts its course 12° southward. After traveling 70 miles in that direction, how far is the ship from its point of departure?

9. $\mathbf{w} = 4\mathbf{u} - 7\mathbf{v}$ where $\mathbf{u} = 3\mathbf{i} + \mathbf{j}$ and $\mathbf{v} = -\mathbf{i} + 2\mathbf{j}$. Find \mathbf{w} .
10. Find a unit vector in the direction of $\mathbf{v} = 5\mathbf{i} - 3\mathbf{j}$.
11. Find the dot product and the angle between $\mathbf{u} = 6\mathbf{i} + 5\mathbf{j}$ and $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$.
12. \mathbf{v} is a vector of magnitude 4 making an angle of 30° with the positive x -axis. Find \mathbf{v} in component form.
13. Find the projection of \mathbf{u} onto \mathbf{v} given $\mathbf{u} = \langle 3, -1 \rangle$ and $\mathbf{v} = \langle -2, 4 \rangle$.

14. Give the trigonometric form of $z = 5 - 5i$.
15. Give the standard form of $z = 6(\cos 225^\circ + i \sin 225^\circ)$.
16. Multiply $[7(\cos 23^\circ + i \sin 23^\circ)][4(\cos 7^\circ + i \sin 7^\circ)]$.

17. Divide $\frac{9\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right)}{3(\cos \pi + i \sin \pi)}$.

18. Find $(2 + 2i)^8$.

19. Find the cube roots of $8\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$.

20. Find all the solutions to $x^4 + i = 0$.

C H A P T E R 7

Linear Systems and Matrices

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